# ROTATORY MOTION

1.   A rod of length *l* is held vertically stationary with its lower end located at a point 'p', on the horizontal plane. When the rod is released to topple about 'P', the velocity of the upper end of the rod with which it hits the ground is :   **(2009 E)**

   1) $\sqrt{\dfrac{g}{l}}$          2) $\sqrt{3gl}$          3) $3\sqrt{\dfrac{g}{l}}$          4) $\sqrt{\dfrac{3g}{l}}$

Ans : 2

Sol:   According to the law of conservation of energy potential energy of the metre stick is converted into rotational kinetic energy

   $\therefore$ Potential energy of metre stick $= \dfrac{mgl}{2}$

   [Since its centre of gravity lies at the middle of the rod]

   $\therefore$ Rotational K.E of a body $= \dfrac{1}{2}Iw^2$

   Where I = moment of Inertia of metre stick about the given point

$$= \dfrac{m\ell^2}{3}$$

   $\therefore$ From law of conservation of energy

$$mg\left(\dfrac{l}{2}\right) = \dfrac{1}{2}I\omega^2$$

$$\Rightarrow mg\left(\dfrac{l}{2}\right) = \dfrac{1}{2}\left(\dfrac{ml^2}{3}\right)\omega^2$$

   But $\omega = \dfrac{V}{r} = \dfrac{V}{l}$

   $\therefore$ On solving V $= \sqrt{3gl}$

2.   A wheel of radius 0.4m can rotate freely about its axis as shown in the figure. A string is wrapped over its rim and a mass of 4kg is hung. An angular acceleration of 8 rad-s$^2$ is produced in it due to the torque. Then moment of inertia of the wheel is:

                                                                                                                          **(2009 E)**

   1) 2 kg-m$^2$          2) 1 kg-m$^2$          3) 4 kg-m$^2$          4) 8 kg-m$^2$

Ans : 1

Sol:   radius = 0.4 m

   Angular acceleration $\alpha = 8 rads^{-2}$

   Mass = 4kg

   Torque ($\tau$) = I$\alpha$       but Torque = Fr = mgr

                       $\Rightarrow mgr = I\alpha \Rightarrow 4 \times 10 \times 0.4 = I \times 8$

                       On simplifying I = 2kg m$^2$

3. A steel wire can withstand a load up to 2940N. A load of 150kg is suspended from a rigid support. The maximum angle through which the wire can be displaced from the mean position, so that the wire does not break when the load passes through the position of equilibrium, is **(2006 E)**

1. $30^0$      2. $60^0$      3. $80^0$      4. $85^0$

Ans:2

Sol: Let the pendulum is displaced through an angle $\theta$ with the vertical.

From conservation of energy

$$\frac{1}{2}mv^2 = mgl[1-\cos\theta]$$

$$V = \sqrt{2gl[1-\cos\theta]}$$

Maximum load the wire can withstand without breaking at the

mean position $= mg + \frac{mv^2}{r} = 2940 \Rightarrow mg + \frac{m}{\ell} \times 2gl[1-\cos\theta] = 2940$

On solving $\theta = 60^0$

4. The moment of inertia of a thin circular disc about an axis passing through its centre and perpendicular to its plane is I. Then, the moment of inertia of the disc about an axis parallel to its diameter and touching the edge of the rim is [2008 E]

1. I      2. 2I      3. $\frac{3}{2}I$      4. $\frac{5}{2}I$

Ans : 4

Sol: Moment of inertia of a thin circular disc about an axis passing through its centre and

perpendicular to its plane $= I = \frac{MR^2}{2}$

$I_1 =$ M.I of the disc about an axis parallel to its diameter and touching the edge of the rim

$$= \frac{MR^2}{4} + MR^2 \text{ [ from parallel axis I = Ig+MR}^2\text{]}$$

$$= \frac{5}{4}MR^2 = \frac{5}{2}I$$

5. A bucket filled with water is tied to a rope of length 0.5m and is rotated in a circular path in vertical plane. the least velocity it should have at the lowest point of circle so that water does not spill is, (g=10ms$^{-2}$) [2007E]

1) $\sqrt{5}m/s$      2) $\sqrt{10}m/s$      3) $5m/s$      4) $2\sqrt{5}m/s$

Ans: 3

Sol: Velocity at lowest point so that water does not spill $= \sqrt{5gr}$

$$= \sqrt{5 \times 10 \times 0.5} = 5\text{ms}^{-1}$$

6. Two solid spheres (A and B) are made of metals of different densities $\rho_A$ and $\rho_B$ respectively. If their masses are equal, the ratio of their moment of inertia $(I_B/I_A)$ about their respective diameter is [2007E]

1) $\left(\dfrac{\rho_B}{\rho_A}\right)^{\frac{2}{3}}$ 　　　　 2) $\left(\dfrac{\rho_A}{\rho_B}\right)^{\frac{2}{3}}$ 　　　　 3) $\dfrac{\rho_A}{\rho_B}$ 　　　　 4) $\dfrac{\rho_B}{\rho_A}$

Ans : 2

Sol:　　As the masses are equal $m_A = m_B$

$$\therefore \frac{4}{3}\pi R_A^3 \rho_A = \frac{4}{3}\pi R_B^3 \rho_B$$

$$\frac{R_A}{R_B} = \left(\frac{\rho_B}{\rho_A}\right)^{\frac{1}{3}} \dots\dots\dots\dots\dots\dots\dots(1)$$

Ratio of moment of Inertia $= \dfrac{I_A}{I_B} = \dfrac{\dfrac{2}{5}m_B R_B^2}{\dfrac{2}{5}m_A R_A^2}$

$$\therefore \frac{I_A}{I_B} = \frac{m_B R_B^2}{m_A R_A^2} \dots\dots\dots\dots\dots\dots\dots(2)$$

Sub (1) in (2)

$$\frac{I_A}{I_B} = \left(\frac{\rho_A}{\rho_B}\right)^{\frac{2}{3}}$$

7.　　A uniform rod of length '8a' and mass '6m' lies on a smooth horizontal surface. two point masses 'm' and '2m' moving in the same plane with speed '2v and v respectively strike the rod perpendicularly at distance 'a' and '2a' from the midpoint of the rod in the opposite directions and stick to the rod. The angular velocity of the system immediately after the collision is: 　　[2006E]

1) $\dfrac{6v}{32a}$ 　　　　 2) $\dfrac{6v}{33a}$ 　　　　 3) $\dfrac{6v}{40a}$ 　　　　 4) $\dfrac{6v}{41a}$

Ans:　4



Sol:
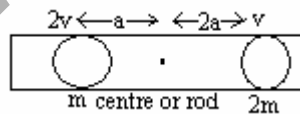
　　According to law of conservation of angular momentum

$$I_1\omega_1 + I_2\omega_2 = I\omega$$

$$\Rightarrow m_1 v_1 r_1 + m_2 v_2 r_2 = I\omega$$

Given $I = \dfrac{Ml^2}{l^2} + ma^2 + 2m(2a)^2$

But M = mass of rod = 6m

$l$ = length of rod = 8a

$$\therefore m(2v)a + 2m(v)(2a) = \left(\frac{6m \times 64a^2}{12} + ma^2 + 8ma^2\right)\omega$$

$$\Rightarrow 2mva + 4mva = (41ma^2)\omega$$

$$\Rightarrow \omega = \frac{6v}{41a}$$

$$\Rightarrow \omega = \frac{6v}{41a}$$

8. Assume the earth's orbit around the sun as circular and the distance between their centres as 'D' Mass of the earth is 'M' and its radius is 'R' If earth has an angular velocity '' with respect to its centre and '' with respect to the centre of the sun, the total kinetic energy of the earth is: [2006E]

1. $\dfrac{MR^2\omega_0^2}{5}\left[1+\left(\dfrac{\omega}{\omega_0}\right)^2+\dfrac{5}{2}\left(\dfrac{D\omega}{R\omega_0}\right)^2\right]$ 

2. $\dfrac{MR^2\omega_0^2}{5}\left[1+\dfrac{5}{2}\left(\dfrac{D\omega}{R\omega_0}\right)^2\right]$

3. $\dfrac{5}{2}MR^2\omega_0^2\left[1+\dfrac{5}{2}\left(\dfrac{D\omega}{R\omega_0}\right)^2\right]$ 

4. $\dfrac{2}{5}MR^2\omega_0^2\left[1+\left(\dfrac{\omega}{\omega_0}\right)^2+\dfrac{5}{2}\left(\dfrac{D\omega}{R\omega_0}\right)^2\right]$

Ans : 2

Sol: Total kinetic energy $=\dfrac{1}{2}\times\dfrac{2}{5}MR^2\omega_0^2+\dfrac{1}{2}MR^2\left(\dfrac{D^2}{R^2}\right)\omega^2$

On simplifying

$$\dfrac{MR^2\omega_0^2}{5}\left[1+\dfrac{5}{2}\left(\dfrac{D\omega}{R\omega_0}\right)^2\right]$$

9. Identify the increasing order of the angular velocities of the following (E-2005)

a) earth rotating about its own axis is

b) hours hand of a clock

c) seconds hand of a clock

d) fly wheel of radius 2m making 300 rps

1) a, b, c, d 　　2) b, c, d, a 　　3) c, d, a, b 　　4) d, a, b, c

Ans : 1

Sol: a) angular velocity of earth rotating about its own axis

$$=\frac{2\pi}{T}=\frac{2\pi}{24\times60\times60}$$
$$=\frac{2\pi}{86400}\,rads^{-1}$$

b) angular velocity of hour's hand

$$=\frac{2\pi}{12\times60\times60}$$
$$=\frac{2\pi}{43200}\,rads^{-1}$$

c) angular velocity of seconds hand $=\dfrac{2\pi}{60}\,rads^{-1}$

d) angular velocity of flywheel

$$=\frac{2\pi}{T}=2\pi\times\frac{30^0}{60^0}$$
$$=10\pi$$

10. A thin uniform square lamina of side 'a' is placed in the X-Y plane with its sides parallel to X and Y-axes and with its centre coinciding with the origin. Its moment of inertia about an axis passing through a point on the Y-axis at a distance $y = 2a$ and parallel to X-axis is equal to its moment of inertia about an axis passing through a point on the X-axis at a distance $x = d$ and perpendicular to X-Y plane. The value of d is **(2004 E)**

1) $\dfrac{7}{3}a$      2) $\sqrt{\dfrac{47}{12}}a$      3) $\dfrac{9}{5}a$      4) $\sqrt{\dfrac{51}{12}}a$

Ans : 2

Sol: From parallel axis theorem $I = I_g + MR^2$

From perpendicular axis theorem $I_z = I_x + I_y$

$$\therefore \frac{Ma^2}{12} + M\left(2a^2\right) = \frac{Ma^2}{6} + Md^2$$

Simplifying we get

$$d = \sqrt{\frac{47}{12}}a$$

11. A particle of mass 1 kg is projected with an initial velocity 10 ms$^{-1}$ at an angle of projection 45$^0$ with the horizontal. The average torque acting on the projectile, between the time at which it is projected and the time at which it strikes the ground, about the point of projection in newton meter is **(2004 E)**

1) 25      2) 50      3) 75      4) 100

Ans: 2

Sol: From the def. $L = mv\,r$

Where r = horizontal distance $= \left(u\cos\theta\right)t$

$V = u\cos\theta$ = horizontal component of velocity

$$\therefore \tau = \frac{dL}{dt} = \frac{d}{dt}\Big[m\left(u\cos\theta\right)\left(u\cos\theta.t\right)\Big] = \frac{d}{dt}\Big[mu^2\cos^2\theta\Big]t$$
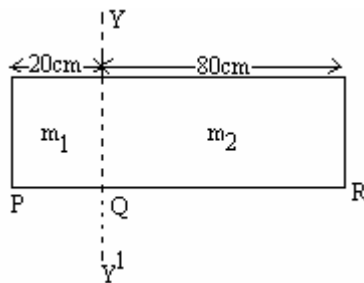
$$\therefore \tau = mu^2\cos^2\theta = 1(10)^2\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 50\ \text{Nm}$$

12. The moment of inertia of a meter scale of mass 0.6 kg about an axis perpendicular to the scale and located at the 20cm position on the scale in kg m$^2$ is : (Breadth of the scale is negligible) **(E-2003)**

1) 0.078      2) 0.104      3) 0.148      4) 0.208

Ans: 1

Sol:

Linear density = mass per unit length = $\dfrac{0.6}{100}$ kg / cm

Mass of part PQ = $m_1$ = linear density $\times$ length of part PQ

$$= \dfrac{0.6}{100} \times 20 = \dfrac{0.6}{5} \text{kg}$$

Mass of part QR = $m_2$ = linear density $\times$ length of part QR

$$= \dfrac{0.6}{100} \times 80 = \dfrac{2.4}{5} \text{kg}$$

$\therefore$ moment of Inertia = $m_1 \left(\dfrac{PQ}{2}\right)^2 + m_2 \left(\dfrac{QR}{2}\right)^2$

Given PQ = $20 \times 10^{-2}$ m , QR = $80 \times 10^{-2}$ m

On substituting we get I = 0.078kg-m$^2$

13. A circular disc of radius R and thickness R/6 has moment of inertia I about an axis passing through its centre and perpendicular to its plane. The disc is melted and recasted into a solid sphere. The moment of inertia of the sphere about its diameter as axis of rotation is : **(E-2003)**

1) I  2) $\dfrac{2I}{3}$  3) $\dfrac{I}{5}$  4) $\dfrac{I}{10}$

Ans: 3

Sol: Moment of inertia of a disc = $I = \dfrac{MR^2}{2}$

As the disc is melted and recasted into a solid sphere volume remains constant.

$\therefore$ Volume of sphere = Volume of disc

$$\dfrac{4}{3}\pi R_1^3 = \pi R^2 \times R / 6$$

On solving $R_1 = R/2$

$\therefore$ moment of Inertia of sphere = $I_1 = \dfrac{2}{5} MR_1^2$

$$\Rightarrow I_1 = \dfrac{2}{5} M \left(\dfrac{R}{2}\right)^2$$

$$\Rightarrow I_1 = \dfrac{2}{5} M \dfrac{R^2}{4}$$

$$\Rightarrow I_1 = \dfrac{1}{5}\left(\dfrac{MR^2}{2}\right) = \dfrac{I}{5}$$

14. A body of mass M kg is on the top point of a smooth hemisphere of radius 5m. It is released to slide down the surface of the hemisphere. It leaves the surface when its

velocity is 5 m/s. At this instant the angle made by the radius vector of the body with the vertical is : (Acceleration due to gravity = 10ms⁻²) **(E-2002)**

1) $30^0$  2) $45^0$  3) $60^0$  4) $90^0$

Ans: 3

Sol: Let the body leaves the surface of the hemisphere at the point P, where the Normal reaction is O.
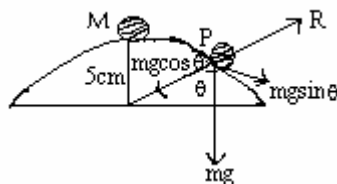


At the point 'P' the radius vector of the body makes and angle $\theta$ with the vertical.

$$\therefore mg\cos\theta - R = \frac{mv^2}{r}$$

As normal reaction = 0

$$\therefore mg\cos\theta = \frac{mv^2}{r} \Rightarrow \cos\theta = \frac{v^2}{rg} = \frac{(5)^2}{5\times10}$$

$$\therefore \cos\theta = \frac{1}{2} \qquad \theta = 60^0$$

15. The diameter of a flywheel is 1 m. It has a mass of 20 kg. It is rotating about its axis with a speed of 120 rotations in one minute. Its angular momentum in kg m²/s is :

(E-2002)

1) 13.4  2) 31.4  3) 41.4  4) 43.4

Ans :2

Sol: $n = \frac{120}{60} = 2rps$

Angular frequency $\omega = 2\pi n = 4\pi rads^{-1}$

Angular momentum $L = I\omega = \left(\frac{MR^2}{2}\right)\omega$

$$= 31.4 \text{ kgm}^2\text{s}^{-1}$$

16. Moment of inertia of a uniform horizontal solid cylinder of mass M about an axis passing through its edge and perpendicular to the axis of the cylinder when its length is 6 times its radius R is : (E-2002)

1) $\frac{39MR^2}{4}$  2) $\frac{39MR}{4}$  3) $\frac{49MR}{4}$  4) $\frac{49MR^2}{4}$

Ans: 4

Sol: From parallel axis theorem

$$I = I_g + MR^2$$

$$\Rightarrow I = M\left[\frac{L^2}{12} + \frac{R^2}{4}\right] + M\left[\frac{L}{2}\right]^2$$

$$\Rightarrow I = \frac{ML^2}{3} + \frac{MR^2}{4}$$

given L = 6R

$$\therefore \ I = \frac{M}{3}(6R)^2 + \frac{MR^2}{4} = \frac{49}{4}MR^2$$

17. A uniform metal rod of length 'L' and mass 'M' is rotating about an axis passing through one of the ends perpendicular to the rod with angular speed '$\omega$'. If the temperature increases by "$t^0C$" then the change in its angular velocity is proportional to which of the following ? (Coefficient of linear expansion of rod $= \alpha$ )    (E-2001)

1) $\sqrt{\omega}$ 　　　　　2) $\omega$ 　　　　　3) $\omega^2$ 　　　　　4) $1/\omega$

Ans: 2

Sol : From the law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

$$\frac{ML_1^2}{12}.\omega_1 = \frac{ML_2^2}{12}.\omega_2$$

$$L_1^2\omega_1 = L_2^2\omega_2 \dots\dots\dots\dots\dots\dots\dots(1)$$

We know that $L_2 = L_1[1+\alpha t]$

Squaring on both sides

$$L_2^2 = L_1^2[1+\alpha t]^2$$

as $\alpha$ is very small

$$\therefore L_2^2 = L_1^2[1+2\alpha t]\dots\dots\dots\dots\dots(2)$$

Sub (2) in (1)

$$\omega_1 = \omega_2[1+2\alpha t]$$

$$\Rightarrow \frac{\omega_2}{\omega_1} = \frac{1}{1+2\alpha t}$$

$$\Rightarrow \frac{\omega_2 - \omega_1}{\omega_1} = \frac{1}{1+2\alpha t} - 1$$

$$\Rightarrow \frac{\Delta\omega}{\omega} = \frac{1-1-2\alpha t}{1+2\alpha t}$$

$$\Rightarrow \Delta\omega \propto \omega$$

18. From a uniform wire, two circular loops are made (i) P of radius 'r' and (ii) Q of radius nr. If the moment of inertia of Q about an axis passing through its centre and perpendicular to its plane is 8 times that of P about a similar axis, the value of 'n' is (diameter of the wire is very much smaller than r or nr)                (E-2001)

1) 8 　　　　　2) 6 　　　　　3) 4 　　　　　4) 2

Ans:4

Sol: Moment of inertia of P = $Mr^2$

Moment of inertia of Q = $(nM) (nr)^2$

[since mass $\propto$ length]

Given $I_Q = 8I_P$

$$\Rightarrow (nM)(n^2r^2) = 8Mr^2$$

$$\Rightarrow n^3 = 8 \Rightarrow n = 2$$

19.    The moment of inertia of a sphere of mass M and radius R about an axis passing through its centre is 2/5 MR$^2$. The radius of gyration of the sphere about a parallel axis to the above and tangent to the sphere is                                    (E-2000)

1) $\dfrac{7}{5}$R          2) $\dfrac{3}{5}$R          3) $\left(\sqrt{\dfrac{7}{5}}\right)$R          4) $\left(\sqrt{\dfrac{3}{5}}\right)$R

Ans :3

Sol:    Moment of Inertia about a tangent parallel to the axis passing through its centre is

From parallel axis theorem

$$I = I_g + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\therefore I = MK^2 = \frac{7}{5}MR^2$$

$$\therefore \text{ Radius of gyration } = K = \left(\sqrt{\frac{7}{5}}\right)R$$

20.    A wheel has a speed of 1200 revolutions per minute and is made to slow down at a rate of 4 radians/s$^2$. The number of revolutions it makes before coming to rest is

(E-2000)

1) 143          2) 272          3) 314          4) 722

Ans : 3

Sol:    $\omega_1 = 2\pi\left[\dfrac{1200}{60}\right] = (40\pi)\text{rads}^{-1}$

$\omega_f = 0$          $\alpha = 4\text{rads}^{-1}$

$$\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{1600\pi^2}{8} = (200\pi^2)\text{rad}$$

$$\therefore \text{ Number of rotation made } = \frac{\theta}{2\pi} \text{ (since } \theta = 2\pi n)$$

$$= 100\pi = 314$$

## MEDICAL

21.    A rod of length $l$ is held vertically stationary with its lower and located at a point 'p', on the horizontal plane. When the rod is released to topple about 'P', the velocity of the upper end of the rod with which it hits the ground is :          **(2009 M)**

1) $\sqrt{\dfrac{g}{l}}$          2) $\sqrt{3gl}$          3) $3\sqrt{\dfrac{g}{l}}$          4) $\sqrt{\dfrac{3g}{l}}$

Ans : 2

Sol:    According to the law of conservation of energy potential energy of the metre stick is converted into rotational kinetic energy

$\therefore$ Potential energy of metre stick $= \dfrac{mgl}{2}$

[Since its centre of gravity lies at the middle of the rod]

$\therefore$ Rotational K.E of a body $= \dfrac{1}{2}Iw^2$

Where I = moment of Inertia of metre stick about the given point

$$= \dfrac{m\ell^2}{3}$$

$\therefore$ From law of conservation of energy

$$mg\left(\dfrac{l}{2}\right) = \dfrac{1}{2}I\omega^2$$

$$\Rightarrow mg\left(\dfrac{l}{2}\right) = \dfrac{1}{2}\left(\dfrac{ml^2}{3}\right)\omega^2 \Rightarrow \omega = \sqrt{\dfrac{3g}{\ell}}$$

But $\omega = \dfrac{V}{r} = \dfrac{V}{l}$

$\therefore$ On solving V$= \sqrt{3gl}$

22. A rigid uniform rod of mass M and length L is resting on a smooth horizontal table. Two marbles each of mass 'm' and traveling with uniform speed 'V' collide with two ends of the rod simultaneously and inelastically as shown. The marbles get struck to the rod after the collision and continue to move with the rod. If m=M/6 and V=L mts/sec, then the time taken by the rod rotate through $\dfrac{\pi}{2}$ is **(2009 E)**
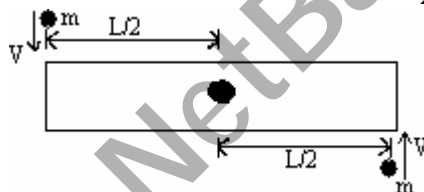


1) 1sec      2) $2\pi$ seconds      3) $\pi$ seconds      4) $\dfrac{\pi}{2}$ seconds

Ans : 4

Sol: According to the law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

$$m_1v_1r_1 + m_1v_1r_1 = I\omega$$

[ where $I = \dfrac{ML^2}{12} + m\left(\dfrac{L}{2}\right)^2 + m\left(\dfrac{L}{2}\right)^2$ ]

$$\Rightarrow mv\left(\dfrac{L}{2}\right) + mv\left(\dfrac{L}{2}\right) = \left[\dfrac{ML^2}{12} + m\left(\dfrac{L}{2}\right)^2 + m\left(\dfrac{L}{2}\right)^2\right]\omega$$

$$\Rightarrow mvL = \left[\dfrac{6mL^2}{12} + \dfrac{mL^2}{4} + \dfrac{mL^2}{4}\right]\omega$$

$$\Rightarrow mvL = \left(\dfrac{12mL^2}{12}\right)\omega$$

$$\Rightarrow \omega = 1 \text{ [since v = L]}$$

$$\Rightarrow T = \dfrac{2\pi}{\omega} = 2\pi s$$

If time period is $2\pi$ seconds then the angle described is $2\pi$ rad.

$$\text{For } \frac{\pi}{2} \text{ rad, time} = \frac{2\pi}{\pi} \times \frac{\pi}{2} = \frac{\pi}{2}\text{s}$$

23. Assertion (A) : A ball connected to a string is in circular motion of a frictionless horizontal table and is in equilibrium.

Reason (R) : Magnitude of the centripetal fore is equal to the magnitude of the tension in the string.

The correct answer is

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true and R is not the correct explanation of A

3) A is true but R is not true

4) A is true but R is true

Ans :4

Sol: A) When a ball is connected to a string which is in circular motion on a frictionless horizontal table is not in equilibrium

B) Magnitude of centripetal force is equal to the magnitude of tension in the string.

24. Circular disc of mass 2kg and radius 1 metre is rotating about an axis perpendicular to its plane and passing through its centre of mass with a rotational kinetic energy of 8 Joules. The angular momentum in (J-sec) is                [2008 M]

1. 8                  2. 4                  3. 2                  4. 1

Ans : 2

Sol: The relation between K.E and angular momentum is $K.E = \frac{L^2}{2I} \Rightarrow L = \sqrt{2(K.E.)I}$

$$I = \text{M.I of disc} = \frac{MR^2}{2} = \frac{2 \times (1)^2}{2} = 1kg - m^2$$
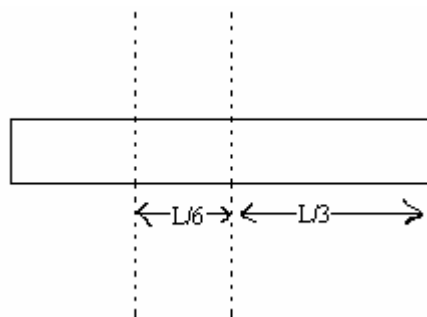$$\therefore L = \sqrt{2(8)(1)} = 4J - s$$

25. The radius of gyration of a rod of length 'L' and mass 'M' about an axis perpendicular to its length and passing through a point at a distance $\frac{L}{3}$ from one of its ends is                [2007M]

1) $\frac{\sqrt{7}}{6}L$          2) $\frac{L^2}{9}$          3) $\frac{L}{3}$          4) $\frac{\sqrt{5}}{2}L$

Ans: 3

Sol: From parallel axis theorem
$$I = I_g + MR^2$$

Where $I_g$= M.I of rod along centre of gravity axis = $\dfrac{ML^2}{12}$

$$MR^2 = M\left[\dfrac{L}{6}\right]^2 = \dfrac{ML^2}{36}$$

$$\therefore I = I_g + MR^2 = \dfrac{ML^2}{12} + \dfrac{ML^2}{36}$$

$$= \dfrac{4ML^2}{36} = \dfrac{ML^2}{9}$$

But $I = MK^2$

$$\therefore MK^2 = \dfrac{ML^2}{9}$$

$$\therefore K = \dfrac{L}{3}$$

26. A ball of mass 0.6kg attached to a light inextensible string rotates in a vertical circle of radius 0.75m such that it has speed of 5ms⁻¹ when the string is horizontal. Tension in the string when it is horizontal on other side is (g-10ms⁻²)  [2007M]

1) 30N 2) 26N 3) 20N 4) 6N

Ans : 3

Sol: Tension in the string when it makes any angle $\theta$ with the vertical

$$T = \dfrac{mv^2}{r} + mg\cos\theta$$

When the string is horizontal then $\theta = 90^0$

$$\therefore T = \dfrac{mv^2}{r} = \dfrac{(0.6)\times(5)}{0.75} = 20N$$

27. The kinetic energy of a body rotating at 300 revolutions per minute is 62.8J. Its angular momentum (in $kgm^2s^{-1}$) is approximately:  [2006M]

1) 1 2) 2 3) 4 4) 8

Ans: 3

Sol: 
$$\omega = 2\pi \times \dfrac{300}{60} = 10\pi rads^{-1}$$

$$\therefore \text{K.E of rotating body} = \dfrac{L\omega}{2}$$

$$\Rightarrow L = \dfrac{2 \times K.E}{\omega}$$

$$= \dfrac{2 \times 62.8}{10\pi} = 4$$

28. A uniform circular disc of radius R lies in the X-Y plane with its centre coinciding with the origin of the coordinate system. Its moment of inertia about an axis, lying in the X-Y plane, parallel to the X-axis and passing through a point on the Y-axis at a distance y=2R is $I_1$. Its moment of inertia about an axis lying in a plane perpendicular to X-Y plane passing through a point on the X-axis at a distance x=d is $I_2$. If $I_1=I_2$, the value of 'd' is                                  [2006M]

1) $\dfrac{\sqrt{19}}{2}R$        2) $\dfrac{\sqrt{17}}{2}R$        3) $\dfrac{\sqrt{15}}{2}R$        4) $\dfrac{\sqrt{13}}{2}R$

Ans:   3

Sol:   $I_1$ = M.I about an axis lying in the x-y plane, parallel to the x-axis and passing through a point on the y-axis at a distance y=2R

$$= \frac{mR^2}{4} + m(2R)^2 = \frac{17mR^2}{4}$$

[from parallel axis theorem]

$I_2$ = M.I about an axis lying in a plane perpendicular to x-y pane passing through a point on the x-axis at a distance x = d

$$= \frac{mR^2}{2} + md^2$$

given that $I_1 = I_2$

$$\therefore \quad \frac{17mR^2}{4} = \frac{mR^2}{2} + md^2$$

On solving d $= \dfrac{\sqrt{15}}{2}R$

29. Assertion (A) : $I_S$ and $I_H$ are the moments of inertia about the diameters of a solid sphere and thin walled hollow sphere respectively. If radii and the masses of the above are equal, then $I_H > I_S$

Reason (R) : In solid sphere , the mass is continuously and regularly distributed about centre , where as the mass , to a large extent concentrated on the surface of hollow sphere                                         (M-2005)

1) Both (A) and (R) are true and (R) is the correct explanation of (A)

2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

3) (A) is true but (B) is false

4) (A) is false but (B) is true

Ans: 1

Sol.   For solid sphere $I_s = \dfrac{2}{5}MR^2$

For Hollow sphere $I_H = \dfrac{2}{3}MR^2$

30. A uniform cylindrical rod of mass M and length L is rotating with an angular speed 'ω'. The axis of rotation is perpendicular to its axis of symmetry and passes through one of its edge faces. If the room temperature increases by 't' and the coefficient of

linear expansion of the rod is '$\alpha$' the magnitude of the change in its angular speed is

(M-2005)

1) $2\omega\alpha t$  2) $\omega\alpha t$  3) $\dfrac{3}{2}\omega\alpha t$  4) $\dfrac{\omega\alpha t}{2}$

Ans: 1

Sol: Since no external torque is acting on the rod, its angular momentum about the axis of rotation should remain constant.

$$\therefore I\omega = I^1\omega^1$$

Where $\omega_1$ is new angular speed and $I^1$ is new moment of inertia of the rod.

$$I = \frac{ML^2}{3}, I^1 = \frac{ML^{1^2}}{3}$$

$$L^2\omega = L^{1^2}\omega^1$$

Here $L_1$ is new length of the rod.

$$L^1 = L(1+\alpha t)$$

So $\omega^1 = \dfrac{\omega}{(1+\alpha t)^2} = \omega(1-2\alpha t)$

(applying Binomial expansion)

Change in angular speed $\Delta\omega = \omega^1 - \omega$

$$= \omega[1-2\alpha t] - \omega = -2\alpha wt$$

31. A uniform circular disc of radius 'R' lies in the X-Y plane with the centre coinciding with the origin. The moment of inertia about an axis passing through a point on the X-axis at a distance x = 2R and perpendicular to the X-Y plane is equal to its moment of inertia about an axis passing through a point on the Y-axis at a distance y = d and parallel to the X-axis in the X-Y plane. The value of 'd' is                 (M-2004)

1) $\dfrac{4R}{3}$  2) $\sqrt{17}\left(\dfrac{R}{2}\right)$  3) $\sqrt{15}\left(\dfrac{R}{2}\right)$  4) $\sqrt{13}\left(\dfrac{R}{2}\right)$

Ans :2

Sol: Applying parallel axis theorem

On x-axis at a distance 2R

$$I = \frac{mR^2}{2} + m(4R^2) = \frac{9}{2}mR^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

On y-axis at a distance d

$$I = \frac{mR^2}{4} + md^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Equation (1) and (2) d = $\sqrt{17}\left(\dfrac{R}{2}\right)$

32. Assertion (A) : If a body moving in a circular path has constant speed, then there is no force acting on it.

Reason (R) : The direction of the velocity vector of a body moving in a circular path is changing                 (M-2004)

1) Both (A) and (R) are true and (R) is the correct explanation of (A)

2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

3) (A) is true but (B) is false

4) (A) is false but (B) is true

Ans: 4

Sol.    1) Centripetal force may exist

2) In circular motion velocity vector changes continuously

33.    Two circular loops A and B of radii $r_A$ and $r_B$ respectively are made from a uniform wire. The ratio of their moments of inertia about axes passing through their centres and perpendicular to their planes is $\dfrac{I_B}{I_A} = 8$ then $\left(\dfrac{r_B}{r_A}\right) =$        (M-2003)

1) 2                    2) 4                    3) 6                    4) 8

Ans :1

Sol:    $I = \dfrac{mr^2}{2} = \dfrac{(2\pi r)\rho(r^2)}{2} = (\pi r^3)\rho$

[mass of the loop = length of loop x linear density]

Where $\rho$ is density

$$\dfrac{I_B}{I_A} = 8 = \dfrac{r_B{}^3}{r_A{}^3} \Rightarrow \dfrac{r_B}{r_A} = 2$$

34.    A particle performs uniform circular motion with an angular momentum L. If the angular frequency f of the particle is doubled, and kinetic energy is halved, its angular momentum becomes :                    (M-2003)

1) 4L                    2) 2 L                    3) $\dfrac{L}{2}$                    4) $\dfrac{L}{4}$

Ans:    4

Sol:    $\text{KE} = \dfrac{1}{2}L\omega$

$\therefore \dfrac{E_1}{E_2} = \dfrac{L_1\omega_1}{L_2\omega_2}$

$\Rightarrow \dfrac{E}{(E/2)} = \dfrac{L}{L_2} \times \dfrac{\omega}{2\omega}$

$\therefore \Rightarrow L_2 = \dfrac{L}{4}$

35.    Two thin uniform circular rings each of radius 10 cm and mass 0.1 Kg are arranged such that they have common centre and their planes are perpendicular to each other. The moment of inertia of this system about an axis passing through their common centre and perpendicular to the plane of one of the rings in kg m$^2$ is : (M-2002)

1) 15 x 10$^{-3}$            2) 5 x 10$^{-3}$            3) 15 x 10$^{-4}$            4) 18 x 10$^{-4}$

Ans :3

Sol:    Moment of inertia $I = mr^2 + \dfrac{mr^2}{2} = \dfrac{3mr^2}{2}$

$$= \dfrac{3}{2}(0.1)(0.1)^2$$

$$= 15 \times 10^{-4}$$

36. The moment of inertia of a uniform disc about an axis passing through its centre and perpendicular to its plane is 1 kg m$^2$. It is rotating with an angular velocity 100 radians / second. Another identical disc is gently placed on it so that their centres coincide. Now these two discs together continue to rotate about the same axis. Then the loss in kinetic energy in kilo joules is : (M-2002)

   1) 2.5           2) 3.0           3) 3.5           4) 4.0

Ans : 1

Sol: Angular velocity of the system

$$\omega = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2} = \frac{1 \times 100 + 1 \times 0}{1 + 1}$$

$$= \frac{(1)(100)}{2} = 50 \text{ rad}^{-1}$$

$\therefore$ Loss of energy

$$= \frac{1}{2} I_1 \omega_1^2 - \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(1)(100)^2 - \frac{1}{2}(2)(50)^2$$

$$= 2500 \text{ J}$$

$$= 2.5 \text{ KJ}$$

37. A particle of mass 'm' is rotating along a circular path of radius 'r'. Its angular momentum is 'L'. The centripetal force acting on the particle is :(M-2002)

   1) $\dfrac{L^2}{mr}$           2) $\dfrac{L^2 m}{r}$           3) $\dfrac{L^2}{mr^2}$           4) $\dfrac{L^2}{mr^3}$

Ans :4

Sol: Angular momentum L = mvr

$$\Rightarrow v = \frac{L}{mr}$$

Centripetal force F = $\dfrac{mv^2}{r} = \dfrac{L^2}{mr^3}$

38. A mass is whirled in a circular path with a constant angular velocity and its angular momentum is L. If the string is now halved keeping the angular velocity the same, the angular momentum is (M-2001)

   1) L/4           2) L/2           3) L           4) 2L

Ans : 1

Sol: We know angular momentum

$$L = I\omega = (mr^2)\omega$$

$$\Rightarrow L \propto r^2 \Rightarrow \frac{L_1}{L_2} = \frac{r_1^2}{r_2^2} = \frac{r^2}{(r/2)^2}$$

$$\Rightarrow \frac{L_1}{L_2} = 4$$

$$\therefore L_2 = \frac{L_1}{4} = \frac{L}{4}$$

39. The angular velocity of a body changes from $\omega_1$ and $\omega_2$ without applying a torque but changing moment of inertia. The ratio of corresponding radii, $r_1 : r_2$ is

**(M-2001)**

1) $\omega_1 : \omega_2$       2) $\sqrt{\omega_1} : \sqrt{\omega_2}$       3) $\sqrt{\omega_2} : \sqrt{\omega_1}$       4) $\omega_2 : \omega_1$

Ans : 3

Sol: According to law of conservation of angular momentum

$$L = I\omega = \left(mr^2\right)\omega$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{r_2^2}{r_1^2}$$

$$\Rightarrow r_1 : r_2$$

$$= \sqrt{\omega_2} : \sqrt{\omega_1}$$

40. A particle tied to a string of negligible weight and length L is rotated in a horizontal circular path with constant angular velocity having time period T. If the string length is shortened by L/2 while the particle is in motion, the time period is    (M-2000)

1) 4T      2) 2T      3) T/2      4) T/4

Ans: 4

Sol: We know $\omega = \dfrac{2\pi}{T}$

According to law of conservation k of angular momentum

$$\Rightarrow mv_1 r_1 = mv_2 r_2$$

$$\Rightarrow r_1^2 \omega_1 = r_2^2 \omega_2 \Rightarrow \frac{r_1^2}{T_1} = \frac{r_2^2}{T_2}$$

$$T_2 = \left[\frac{r_2^2}{r_1^2}\right] T_1 = \left[\frac{\left(L^2/4\right)}{L^2}\right] T = \frac{T}{4}$$

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