ELEMENTS OF VECTORS

PREVIOUS EAMCET BITS

1. Two persons A and B are located in X-Y plane at the points (0, 0) and (0, 10) respectively. (The distances are measured in M.K.S units). At a time t = 0, they start moving simultaneously with velocities $\overline{v}_A = 2\hat{j}ms^{-1}$ and $\overline{v}_B = 2\hat{i}\,ms^{-1}$ respectively. The time after which A and B are at their closest distance is **[EAMCET 2009 E]**

   1) 2.5 s 2) 4s 3) 1 s 4) $\frac{10}{\sqrt{2}}s$

   Ans: 1

Sol: Resultant velocity = $\sqrt{V_A^2 + V_B^2} = 2\sqrt{2}ms^{-1}$

Resultant displacement = $10\sqrt{2}m$

The time after which both A and B are at their closest distance is

$$\text{time} = \frac{\text{displacement}}{\text{velocity}} = \frac{\frac{10\sqrt{2}}{2}}{2\sqrt{2}} = 2.5\,s$$

2. The component of vector $\overline{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ along the direction of $\hat{i} - \hat{j}$ is **[EAMCET 2008 E]**

   1) $a_x - a_y + a_z$ 2) $a_x - a_y$ 3) $\frac{a_x - a_y}{\sqrt{2}}$ 4) $(a_x + a_y + a_z)$

   Ans: 3

Sol: Component of $a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ along the direction of $\hat{i} - \hat{j}$ is

$$= \frac{(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}).(\hat{i} - \hat{j})}{\sqrt{(1)^2 + (-1)^2}} = \frac{a_x - a_y}{\sqrt{2}}$$

Formula : If $\vec{A}$ and $\vec{B}$ are two vectors then the (i) component of $\vec{A}$ a long $\vec{B}$ is

$$A\cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{B}|}$$

(ii) component of $\vec{B}$ along $\vec{A}$ is $B\cos\theta = \frac{\vec{A} - \vec{B}}{|\vec{A}|}$

3. Velocity and acceleration vectors of charged particle moving perpendicular to the direction of a magnetic field at a given time are $\vec{v} = 2\hat{i} + c\hat{j}$ and $\vec{a} = 3\hat{i} + 4\hat{j}$ respectively, then the value of 'c' is **[EAMCET 2007 E]**

   1) 3 2) 1.5 3) – 1.5 4) –3

   Ans: 3

Sol: In the problem it is given that acceleration and velocity are perpendicular.

$\therefore$ We know that dot product between two perpendicular vectors is zero.

$\therefore$ $\overline{a}.\overline{v} = 0$

$\therefore (3\hat{i} + 4\hat{j}).(2\hat{i} + c\hat{j}) = 0$

$\therefore 6+4c=0 \Rightarrow c=\frac{-6}{4}=-1.5$

4. When a man is standing, rain drops appear to him falling at 60° from the horizontal from his front side. When he is travelling at 5 km per hour on a horizontal road they appear to him falling at 30° , from the horizontal from his front side. The actual speed of the rain in (in km per hour) **[EAMCET 2006E]**

1) 3 2) 4 3) 5 4) 6

Ans: 3

Sol: From parallelogram law of vectors

$$\tan\alpha=\frac{Q\sin\theta}{P+Q\cos\theta} \Rightarrow \frac{1}{\sqrt{3}}=\frac{V_R\sin 60°}{V_m+V_R\cos 60°}$$

Where V_r is the velocity of rain and V_m is the velocity of man

$$\therefore \Rightarrow \frac{1}{\sqrt{3}}=\frac{V_R\times\frac{\sqrt{3}}{2}}{5+V_R\times\frac{1}{2}}$$

On solving V_R = 4 kmph

5. At a given instant of time the position vector of a particle moving in a circle with a velocity $3\hat{i}-4\hat{j}+5\hat{k}$ is $\hat{i}+9\hat{j}-8\hat{k}$. Its angular velocity at that time is **[EAMCET 2005 E]**

1) $\frac{\left(13\hat{i}-29\hat{j}-31\hat{k}\right)}{\sqrt{146}}$ 2) $\frac{\left(13\hat{i}-29\hat{j}-31\hat{k}\right)}{146}$ 3) $\frac{\left(13\hat{i}+29\hat{j}-31\hat{k}\right)}{\sqrt{146}}$ 4) $\frac{\left(13\hat{i}+29\hat{j}+31\hat{k}\right)}{146}$

Ans: 2

Sol: From the relation $\vec{v}=\vec{\omega}\times\vec{r}$

$\vec{v}$ should be perpendicular to $\vec{\omega}$ and $\vec{r}$. So check the condition $\vec{v}.\vec{\omega}=0$, $\vec{\omega}.\vec{r}=0$

[since the dot product of any two perpendicular vectors is zero]

On verifying the above conditions the answer is 2 option

6. At a given instant of time two particles are having the position vectors $4\bar{i}-4\bar{j}+7\bar{k}$ metres and $2\bar{i}+2\bar{j}+5\bar{k}$ meters respectively. If the velocity of the first particle be $0.4\bar{i}\ ms^{-1}$, the velocity of second particle in metre per second if they collide after 10 sec is **[EAMCET 2004E]**

1) $6\left(\bar{i}-\bar{j}+\frac{1}{3}\bar{k}\right)$ 2) $0.6\left(\bar{i}-\bar{j}+\frac{1}{3}\bar{k}\right)$ 3) $6\left(\bar{i}+\bar{j}+\frac{1}{3}\bar{k}\right)$ 4) $0.6\left(\bar{i}+\bar{j}-\frac{1}{3}\bar{k}\right)$

Ans : 2

Sol: As both the particles are colliding the distances travelled are same

$$\vec{s}_1+\vec{u}_1t=\vec{s}_2+\vec{u}_2t$$

$$\Rightarrow\left(4\hat{i}-4\hat{j}+7\hat{k}\right)+\left(0.4\hat{i}\right)=\left(2\hat{i}+2\hat{j}+5\hat{k}\right)+u_2(10)$$

$$\Rightarrow 4\hat{i}-4\hat{j}+7\hat{k}+4\hat{i}=2\hat{i}+2\hat{j}+5\hat{k}+10\vec{u}_2$$

$$\Rightarrow 6\hat{i}-6\hat{j}+2\hat{k}=10\vec{u}_2$$

$$\Rightarrow \vec{u}_2=0.6\left(\hat{i}-\hat{j}+\frac{\hat{k}}{3}\right)$$

7. Two particles having position vectors $\vec{r}_1 = \left(3\vec{i} + 5\vec{j}\right)$ metre and $\vec{r}_2 = \left(-5\vec{i} - 3\vec{j}\right)$ metres are moving with velocities $\vec{r}_1 = \left(4\vec{i} + 3\vec{j}\right)$ m/s and $\vec{r}_2 = \left(a\vec{i} + 7\vec{j}\right)$ m/s. If they collide after 2 seconds. The value of 'a' is **[EAMCET 2003]**

1) 2 2) 4 3) 6 4) 8

Ans: 4

Sol: If two particles collide after a time of 't' then they travel equal distances

$\therefore\ \vec{r}_1 + \vec{v}_1 t = \vec{r}_2 + \vec{v}_2 t$

$\left(3\hat{i} + 5\hat{j}\right) + \left(4\hat{i} + 3\hat{j}\right)2 = \left(-5\hat{i} + 3\hat{j}\right) + \left(-a\hat{i} + 4\hat{j}\right)2$

On solving a = 8

8. A proton of velocity $\left(3\overline{i} + 2\overline{j}\right)$ ms^{-1} enters a field of magnetic induction $\left(2\overline{j} + 3\overline{k}\right)$ Tesla. The acceleration produced in the proton in ms^{-2} is (Specific charge of proton = $0.96\times10^{-8}\,Ckg^{-1}$) **[EAMCET 2002]**

1) $0.96\times10^{8}\left(6\hat{i} + 4\hat{k} + 9\hat{j}\right)$ 2) $0.96\times10^{8}\left(6\hat{i} - 9\hat{j} - 4\hat{k}\right)$

3) $0.96\times10^{8}\left(\hat{i} - \hat{j} - \hat{k}\right)$ 4) $0.96\times10^{8}\left(5\hat{i} - 9\hat{j} - 4\hat{k}\right)$

Ans: 2

Sol: From the relation $\vec{F} = q\left[\vec{V}\times\vec{B}\right]$

$ma = q\left[\vec{V}\times\vec{B}\right]$

$\therefore a = \dfrac{q}{m}\left[\vec{V}\times\vec{B}\right]$

$$\therefore\ \vec{V}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 2 & 0 & 3 \end{vmatrix} = \left(6\hat{i} - 9\hat{j} - 4\hat{k}\right)$$

$\therefore\ a = 0.96\times10^{8}\left[6\hat{i} - 9\hat{j} - 4\hat{k}\right]$

9. An electron moves with speed 2 x 10^5 m/s along the positive x-direction in the presence of a magnetic induction $\overline{B} = \overline{i} + 4\overline{j} - 3\overline{k}$ (in tesla). The magnitude of the force experienced by the electron in Newtons is (Charge on the electron = $1.6\times10^{-19}\,C$) **[EAMCET 2001 E]**

1) 1.18×10^{-13} 2) 1.28×10^{-13} 3) 1.6×10^{-13} 4) 1.72×10^{-13}

Ans: 3

Sol: $\vec{F} = q\left[\vec{V}\times\vec{B}\right]$

$F = 1.6\times10^{-19}\left[\left(2\times10^{5}\hat{i}\right)\times\left(\hat{i} + 4\hat{j} - 3\hat{k}\right)\right]$

$= 1.6\times10^{-19}\left[\left(8\times10^{5}\hat{k} + 6\times10^{5}\hat{j}\right)\right]$

$\therefore \left|\vec{F}\right| = 1.6\times10^{-19}\times10^{6} = 1.6\times10^{-13}\,N$

10. The displacement $\overline{r}$ of a charge Q in an electric field $\overline{E} = e_1\overline{i} + e_2\overline{j} + e_3\overline{k}$ is $\overline{r} = a\overline{i} + b\overline{j}$. The work done is **[EAMCET 2000 E]**

1) $Q\left(ae_1 + be_2\right)$ 2) $Q\sqrt{\left(ae_1\right)^2 + \left(be_2\right)^2}$ 3) $Q\left(e_1 + e_2\right)\sqrt{a^2 + b^2}$ 4) $Q\left(e_1 + e_2\right)\sqrt{a^2 + b^2}$

Ans: 1

Sol: Work done $= \bar{F}.\bar{S} = \bar{E}q.\bar{S} = q\left[\bar{E}.\bar{S}\right]$

$= Q\left[e_1\hat{i} + e_2\hat{j} + e_3\hat{k}\right]\left(a\hat{i} + b\hat{j}\right)$

$= Q\left[e_1 a + e_2 b\right] = Q\left[ae_1 + be_2\right]$

MEDICAL

11. Two persons A and B are located in X-Y plane at the points (0, 0) and (0, 10) respectively. (The distances are measured in M.K.S units). At a time t = 0, they start moving simultaneously with velocities $\bar{v}_A = 2\hat{j}ms^{-1}$ and $\bar{v}_B = 2\hat{i}\,ms^{-1}$ respectively. The time after which A and B are at their closest distance is **[EAMCET 2009 M]**

1) 2.5 s 2) 4s 3) 1 s 4) $\frac{10}{\sqrt{2}}s$

Ans: 1

Sol: Resultant velocity = $\sqrt{V_A^2 + V_B^2} = 2\sqrt{2}ms^{-1}$

Resultant displacement = $10\sqrt{2}m$

The time after which both A and B are at their closest distance is

$$\text{time} = \frac{\text{displacement}}{\text{velocity}} = \frac{\frac{10\sqrt{2}}{2}}{2\sqrt{2}} = 2.5s$$

12. Three forces $\vec{A} = \left(\hat{i} + \hat{j} + \hat{k}\right)$, $\vec{B} = \left(2\hat{i} + \hat{j} + 3\hat{k}\right)$ and $\vec{C}$ acting on a body to keep it in equilibrium. Then $\vec{C}$ is **[EAMCET 2008 M]**

1) $-\left(3\hat{i} + 4\hat{k}\right)$ 2) $-\left(4\hat{i} + 3\hat{k}\right)$ 3) $3\hat{i} + 4\hat{j}$ 4) $2\hat{i} + 3\hat{k}$

Ans: 1

Sol: If three forces are acting on a body and keep it in equilibrium then $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$\therefore\ \vec{A} + \vec{B} + \vec{C} = 0$

On solving $\vec{C} = -\left(3\hat{i} + 4\hat{k}\right)$

13. Given two vectors $\bar{A} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\bar{B} = 4\hat{i} - 2\hat{j} + 6\hat{k}$. The angel made by $\left(\bar{A} + \bar{B}\right)$ with X-axis is **[EAMCET 2007 M]**

1) 30° 2) 45° 3) 60° 4) 90°

Ans: 2

Sol: $\vec{A} + \vec{B} = 3\hat{i} + 3\hat{k}$ and unit vector along x-axis is $\hat{i}$

$\therefore\ \cos\theta = \frac{\left(3\hat{i} + 3\hat{k}\right).\left(\hat{i}\right)}{\sqrt{3^2 + 3^2}\sqrt{1^2}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}} = 45°$

14. Of the vectors given below, the parallel vectors are ,
$\vec{A} = 6\hat{i} + 8\hat{j}$, $\vec{B} = 210\hat{i} + 280\hat{j}$, $\vec{C} = 5.1\hat{i} + 6.8\hat{j}$, $\vec{D} = 3.6\hat{i} + 8\hat{j} + 48\hat{k}$ **[EAMCET 2006 M]**

1) $\vec{A}$ and $\vec{B}$ 2) $\vec{A}$ and $\vec{C}$ 3) $\vec{A}$ and $\vec{D}$ 4) $\vec{C}$ and $\vec{D}$

Ans: 2

Sol: Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel if $\frac{\vec{A}}{\vec{B}}$ = constant

$\therefore \vec{C} = 5.1\hat{i} + 6.8\hat{j} = \frac{17}{20}\left[6\hat{i} + 8\hat{j}\right] = \frac{17}{20}\left(\vec{A}\right)$

$\therefore \vec{A}$ and $\vec{C}$ are parallel

15. Angle (in rad) made by the vector $\sqrt{3}\,\hat{i} + \hat{j}$ with the x-axis **[EAMCET 2005 M]**

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: 1

Sol: Angle made by the vector $\vec{A} = A_x\hat{i} + A_y\hat{j}$ with x- axis is

$\tan\alpha = \frac{A_y}{A_x} = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \frac{\pi}{6}$

16. Two vector $\bar{Q}$ which has a magnitude of 8 is added to the vector $\bar{P}$ which lies along the x-axis. The resultant of these two vectors is a third vector $\bar{R}$ which lies along the y-axis and has a magnitude twice that of $\bar{P}$. The magnitude of $\bar{P}$ is **[EAMCET 2004 M]**

1) $\frac{6}{\sqrt{5}}$ 2) $\frac{8}{\sqrt{5}}$ 3) $\frac{12}{\sqrt{5}}$ 4) $\frac{16}{\sqrt{5}}$

Ans: 2

Sol: In the problem Q = 8 units ………(1)

$\vec{R} = 2\vec{P}\left[\text{given}\right]$ …………...(2)

Since $\vec{P}$ is along x-axis and $\vec{R}$ is along y-axis

$\therefore$ $\vec{P}$ and $\vec{R}$ are perpendicular vectors

$\therefore$ $Q^2 = P^2 + R^2$ ……………(3)

Sub (1) and (2) in (3) $P = \frac{8}{\sqrt{5}}$

17. A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off at right angles to each other, one with a velocity $2\bar{i}\,m/s$ and the other with a velocity $3\bar{j}\,m/s$. If the explosion takes place in 10^{-5} sec. The average force acting on the third piece in Newton is **[EAMCET 2003 M]**

1) $\left(2\vec{i} + 3\vec{j}\right)\times 10^{-5}$ 2) $-\left(2\vec{i} + 3\vec{j}\right)\times 10^{5}$ 3) $\left(3\vec{j} + 2\bar{i}\right)\times 10^{5}$ 4) $\left(2\vec{i} + 3\vec{j}\right)\times 10^{-5}$

Ans: 2

Sol: According to the law of conservation of momentum

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0 \Rightarrow \vec{P}_3 = -\left(\vec{P}_1 + \vec{P}_2\right)$

$= -\left(2\hat{i} + 3\hat{j}\right)$

$F = \frac{dp}{dt}\left[\text{ Newtons II}^{nd}\text{ law}\right]$

$= \frac{-\left(2\hat{i} + 3\hat{j}\right)}{10^{-5}} = -\left(2\hat{i} + 3\hat{j}\right)\times 10^{5}$

18. A boat which has a speed of 13 kmph in still water crosses a river of width 1 km along the shortest possible path in 12 minutes. The velocity of the river water in kmph is

[EAMCET 2002M]

1) 12 2) 10 3) 8 4) 6

Ans: 1

Sol: Time taken to cross the river in shortest possible path is = $\frac{\text{width of river}}{\sqrt{v_b^2 - v_r^2}}$

$\frac{12}{60} = \frac{1}{\sqrt{169 - v_r^2}} \Rightarrow v_r = 12\,\text{kmph}$

19. The unit vector parallel to the resultant of the vectors $\vec{A} = 4\vec{i} + 3\vec{j} + 6\vec{k}$ and $\vec{B} = -\vec{i} + 3\vec{j} - 8\vec{k}$ is

[EAMCET 2000 M]

1) $\frac{1}{7}\left[3\vec{i} + 6\vec{j} - 2\vec{k}\right]$ 2) $\frac{1}{7}\left[3\vec{i} + 6\vec{j} + 2\vec{k}\right]$ 3) $\frac{1}{49}\left[3\vec{i} + 6\vec{j} + 2\vec{k}\right]$ 4) $\frac{1}{49}\left[3\vec{i} + 6\vec{j} - 2\vec{k}\right]$

ans: 1

Sol: The unit vectors parallel to the resultant of $\bar{A}$ and $\bar{B}$ is $\frac{\left(\bar{A} + \bar{B}\right)}{\left|\bar{A} + \bar{B}\right|}$

Resultant of $\vec{A}$ and $\vec{B} = \vec{A} + \vec{B}$

$\vec{A} + \vec{B} = 3\hat{i} + 6\hat{j} - 2\hat{k}$

$\therefore$ Unit vector = $\frac{\vec{A} + \vec{B}}{\left|\vec{A} + \vec{B}\right|} = \frac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7} = \frac{1}{7}\left[3\hat{i} + 6\hat{j} - 2\hat{k}\right]$

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