CENTRE OF MASS

PREVIOUS EAMCET QUESTIONS

ENGINEERING

1. Two particles A and B initially at rest, move towards each other, under mutual force of attraction. At an instance when the speed of A is 'v' and speed of B is '2v', the speed of centre of mass (c.m) is **(2008 E)**

1) Zero 2) v 3) 2.5v 4) 4v

Ans : 1

Sol: Internal forces do not change the C.M.

$\therefore$ Velocity or speed of centre of mass is zero

2. Two bodies of 6kg and 4kg masses have their velocity $5\hat{i}-2\hat{j}+10\hat{k}$ and $10\hat{i}-2\hat{j}+5\hat{k}$ respectively. Then the velocity of their centre of mass is **(2007 E)**

1) $5\hat{i}+2\hat{j}-8\hat{k}$ 2) $7\hat{i}+2\hat{j}-8\hat{k}$ 3) $7\hat{i}-2\hat{j}+8\hat{k}$ 4) $5\hat{i}-2\hat{j}+8\hat{k}$

Ans : 3

Sol: $m_1 = 6kg$, $m_2 = 4kg$

$\overline{V_1} = 5\hat{i}-2\hat{j}+10\hat{k}$, $\overline{V_2} = 10\hat{i}-2\hat{j}+5\hat{k}$

Velocity of centre of mass = $\overrightarrow{V_{C.M}} = \dfrac{m_1\overrightarrow{v_1}+m_2\overrightarrow{v_2}}{m_1+m_2}$

$= 7\hat{i}+2\hat{j}-8\hat{k}$

3. The centre of mass of three particle s of masses 1kg, 2kg, and 3kg is at (2, 2, 2). The position of the fourth mass of 4kg to be placed in the system so that the new centre of mass is at (0, 0, 0) is **(2005 E)**

1. (–3, –3, –3) 2. (–3, 3, –3) 3. (2, 3, –3) 4. (2, –2, 3)

Ans:1

Sol: As the c.m. of three particles is at (2,2,2)

$\therefore$ The total mass = 1+2+3 = 6kg

Now consider the 4kg mass at the position (x,y,z)

Now centre of mass of total system at (0,0,0)

$\therefore \dfrac{6\times 2+4x}{10} = 0 \Rightarrow$ 12 = -4x $\Rightarrow$ x = -3

Similarly $\dfrac{6\times 2+4y}{12} = 0 \Rightarrow$ y = -3

$\dfrac{6\times 2+4z}{12} = 0 \Rightarrow$ z = -3

From the above we can conclude that

x = -3, y = -3, z = -3

4. Two particles of equal mass have velocities $\bar{V}_1 = 4\bar{i}$ and $V_2 = 4\bar{j}$. First particle has an acceleration $\bar{a}_1 = (5\bar{i} + 5\bar{j})\, ms^{-2}$ while the acceleration of the other particle is zero. The centre of mass of the two particles moves in a path of **[2004 E]**

1. Straight line 2. Parabola 3. Circle 4. Ellipse

Ans : 1

Sol: $$\overrightarrow{V_{C.M}} = \frac{m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}}{m_1 + m_2} = \frac{m\left[4\hat{i} + 4\hat{j}\right]}{2m} = 2\left[\hat{i} + \hat{j}\right]$$

$$\overrightarrow{a_{C.M}} = \frac{m_1\overrightarrow{a_1} + m_2\overrightarrow{a_2}}{m_1 + m_2} = \frac{m\left[5\hat{i} + 5\hat{j}\right] + m \times 0}{2m}$$

$$= \frac{5}{2}\left[\hat{i} + \hat{j}\right]$$

Both $\overrightarrow{V_c}$ and $\overrightarrow{a_c}$ are in the same direction

Hence the centre of mass moves in a straight line.

5. Two objects of masses 200gm and 500gm have velocities of 10i m/s and 3i + 5j m/s respectively. The velocity of their centre of mass is **[2003 E]**

1) $5\hat{i} - 25\hat{j}$ 2) $\frac{5}{7}\hat{i} - 25\hat{j}$ 3) $5\hat{i} + \frac{25}{7}\hat{j}$ 4) $25\hat{i} - \frac{5}{7}\hat{j}$

Ans : 3

Sol: $$\overrightarrow{V_{C.M}} = \frac{m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2}}{m_1 + m_2}$$

$$= \frac{200\left(10\hat{i}\right) + 500\left(3\hat{i} + 5\hat{j}\right)}{200 + 500}$$

$$= \frac{5}{7}\hat{i} + 25\hat{j}$$

6. Particles of masses m, 2m, 3m .... nm grams are placed on the same line at distances, l, 2l, 3l, ... nl cm from a fixed point. The distance of centre of mass of the particles from the fixed point in centimeters in **[2002 E]**

1) $\frac{(2n+1)l}{3}$ 2) $\frac{l}{n+1}$ 3) $\frac{n(n^2+l)l}{2}$ 4) $\frac{2l}{n(n^2+l)l}$

Ans : 1

Sol: Distance of centre of mass

$$x_{c.m} = \frac{m_1x_1 + m_2x_2 + ...m_nx_n}{m_1 + m_2 + ...m_n}$$

$$= \frac{ml + (2m)(2l) + ...(nm)(nl)}{m + 2m + ... + nm}$$

$$= \frac{ml\left[1 + 4 + 9 + ... + n^2\right]}{m\left[1 + 2 + ... + n\right]}$$

We know that $1^2 + 2^2 + 3^2 + ... + n^2 = \Sigma n^2$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$1 + 2 + 3 + \ldots\ldots + n = \Sigma n = \frac{n(n+1)}{2}$$

$$\therefore x_{c.m} = \frac{\frac{\ell n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} = \frac{l(2n+1)}{3}$$

7. The velocities of three particles of masses 20g, 30g and 50g are $10\bar{i}$, $10\bar{j}$ and $10\bar{k}$ respectively. The velocity of the centre of mass of the three particles is **(2001 E)**

1) $2\bar{i}+3\bar{j}+5\bar{k}$ 2) $10(\bar{i}+\bar{j}+\bar{k})$ 3) $20\bar{i}+30\bar{j}+5\bar{k}$ 4) $2\bar{i}+30\bar{j}+50\bar{k}$

Ans: 1

Sol:

$$\overline{V_{C.M}} = \frac{m_1\vec{v_1}+m_2\vec{v_2}+m_3\vec{v}_3}{m_1+m_2+m_3}$$

$$= \frac{20\times10\hat{i}+30\times10\hat{j}+50\times10\hat{k}}{20+30+50}$$

$$= 2\bar{i}+3\bar{j}+5\bar{k}$$

MEDICAL

8. Four particles, each of mass 1kg are placed at the corners of a square OABC of side 1m. O is the origin of the coordinate system. OA and OC are aligned along positive X-axis and positive Y-axis respectively. The position vector of the centre of mass is (in 'm') **(2006 M)**

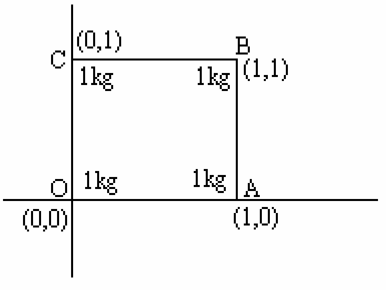
1) $\hat{i}+\hat{j}$ 2) $\frac{1}{2}(\hat{i}+\hat{j})$ 3) $(\hat{i}-\hat{j})$ 4) $\frac{1}{2}(\hat{i}-\hat{j})$

Ans : 2

Sol:

$$x_{c.m} = \frac{m_1x_1+m_2x_2+m_3x_3+m_4x_4}{m_1+m_2+m_3+m_4}$$

$$y_{c.m} = \frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4}{m_1+m_2+m_3+m_4}$$



$\therefore$ By substituting the coordinates we get

$$x_{c.m} = \frac{1}{2}\hat{i},\ y_{c.m} = \frac{1}{2}\hat{j}$$

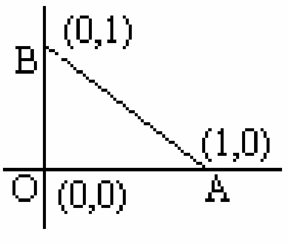
$\Rightarrow$ Position of c.m. = $\frac{1}{2}\left(\hat{i}+\hat{j}\right)$

9. Three particles each 1kg mass are placed at the corners of a right angled triangle AOB, O being the origin of the co–ordinate system (OA and OB) along +ve x-direction and +ve y – direction. If the positive vector of the centre of OA = OB = 1m (in meters) **(2005 M)**

1) $\frac{i+j}{3}$ 2) $\frac{i-j}{3}$ 3) $\frac{2(i+j)}{3}$ 4) (i – j)

Ans : 1

Sol: Given OA=OB=1m, $m_1 = m_2 = m_3 = 1$kg



Given $x_1 = 0$, $x_2 = 1$, $x_3 = 0$

$y_1 = 0$, $y_2 = 0$, $y_3 = 1$

$$x_{c.m} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{1}{3}$$

$$y_{c.m} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{1}{3}$$

$\therefore$ Position vector of c.m. = $x_{c.m}\hat{i} + y_{c.m}\hat{j}$

$= \frac{i+j}{3}$

10. Four particles, each of mass 1kg, are placed at the corners of a square of side one meter in the X–Y plane. If the point of intersection of the diagonals of the square is taken as the origin, the coordinates of the centre of mass are **(2004 M)**

1. (1, 1) 2. (–1, 1) 3. (1, –1) 4. (0, 0)

Ans: 4

Sol: For a square centre of mass is at the point of intersection of diagonals. If this point is taken at the origin then co-ordinates of the centre of mass is (0,0)

11. One end of a thin uniform rod of length L and mass M_1 is rivetted to the centre of a uniform circular disc of radius 'r' and mass M_2 so that both are coplanar. The centre of mass of the combination from the centre of the disc is (assume that the point of attachment is at the origin) **(2005 M)**

1) $\frac{L(M_1 + M_2)}{2M_1}$ 2) $\frac{LM_1}{2(M_1 + M_2)}$ 3) $\frac{2(M_1 + M_2)}{LM_1}$ 4) $\frac{2LM_1}{(M_1 + M_2)}$

Ans: 2

Sol: The length of the rod = L

$\therefore x_1 = \frac{L}{2}$, $x_2 = 0$ (since c.m of the disc is at the origin)

$$\therefore x_{c.m} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{M_1 \times \frac{L}{2} + M_2 \times 0}{M_1 + M_2}$$

$$= \frac{L}{2}\left[\frac{M_1}{M_1 + M_2}\right]$$

12. A system consists of two identical particles. One particle is at rest and the other particle has an acceleration ‘a’. The centre of mass of the system has an acceleration of **(2002 M)**

1) 2a 2) a 3) $\frac{a}{2}$ 4) $\frac{a}{4}$

Ans:4

Sol: Acceleration of centre of mass of the system

$$a_{CM} = \frac{m_1a_1 + m_2a_2}{m_1 + m_2}$$

$$= \frac{ma}{2m}$$

$$= \frac{a}{2}$$

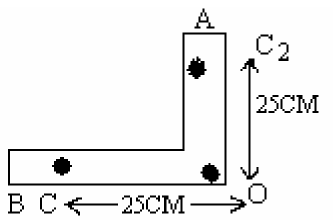
13. A uniform rod of length one meter is bent at its midpoint to make 90^0 angle. The distance of the centre of mass from the centre of the rod is **(2001 M)**

1) 36.1 cm 2) 25.2 cm 3) 17.7 cm 4) zero

Ans: 4

Sol: As the rod is bent a midpoint, both the parts have equal mass.

The distance from the point ‘O’ to the centres of mass of OA and OB are 25 cm, 25cm respectively.



$$\therefore C_1C_2 = \sqrt{(25)^2 + (25)^2} = 25\sqrt{2}$$

$$= 35.35 \text{ cm}$$

$\therefore$ Distance to the effective centre of mass

$$= \frac{35.35}{2} = 17.67\,cm = 17.7 \text{ (nearly)}$$

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