2. THEORY OF EQUATIONS

PREVIOUS EAMCET Bits

EAMCET-2001

1. Each of the roots of the equation $x^3-6x^2+6x-5=0$ are increased by k so that the new transformed equation does not contain term. Then k =

   1. $-\frac{1}{3}$ 2. $-\frac{1}{2}$ 3. -1 4. -2

   Ans: 4

Sol. The transformed equation is $(x-k)^3-6(x-k)^2+6(x-k)-5=0$

Coefficient of x^2 is 0 $\Rightarrow -3k-6=0$ $\Rightarrow k=-2$

2. The roots of the equation $x^3-14x^2+56x-64=0$ are in .... progression.

   1. Arithmetico-geometric 2. Harmonic 3. Arithmetic 4. Geometric

   Ans : 4

Sol. By verification x = 2 is a factor of given equation.

2	1	−14	56	−64
	0	2	−24	64
	1	−12	32	0

(x-2) $(x^2-12x+32)=0$

$x^2-12x+32=0$

x = 4,8

$\therefore$ Roots are 2,4,8

$\therefore$ These are in G.P.

3. If there is a multiple root of order 3 for the equation $x^4-2x^3+2x-1=0$, then the other root is

   1. -1 2. 0 3. 1 4. 2

   Ans: 1

Let f(x) = x^4-2x^3+2x-1 $\Rightarrow f(1)=0$

$\Rightarrow f'(x)=4x^3-6x^2+2 \Rightarrow f'(1)=0$

$\Rightarrow f^{11}(x)=12x^2-12x \Rightarrow f^{11}(1)=0$

Roots of given equation are 1,1,1

Let the other root be α

$S_1=2$

$1+1+1+\alpha$

$\alpha=-1$

$\therefore$ Other root is -1

4. The equation whose roots are the negatives of the roots of the equation $x^7+3x^5+x^3-x^2+7x+2=0$

   1. $x^7+3x^5+x^3+x^2-7x+2=0$ 2. $x^7+3x^5+x^3+x^2+7x-2=0$

3. $x^7+3x^5+x^3-x^2-7x-2=0$ 4. $x^7+3x^5+x^3-x^2+7x-2=0$

Ans: 2

Sol. f(-x) = 0

$(-x)^7 + 3(-x)^5 +(-x)^3 -(-x)^2 + 7(-x) + 2 = 0$

$-x^7-3x^5-x^3-x^2-7x+2=0$

$x^7+3x^5+x^3+x^2+7x-2=0$

5. The biquadratic equation, two of whose roots are 1 + i, $1-\sqrt{2}$ is

1. $x^4-4x^3+5x^2-2x-2=0$ 2. $x^4-4x^3-5x^2+2x+2=0$

3. $x^4+4x^3-5x^2+2x-2=0$ 4. $x^4+4x^3+5x^2-2x+2=0$

Ans: 1

Sol. The roots of required equation are

1+i, 1-i, $1-\sqrt{2}$, $1+\sqrt{2}$

Here S_1 = $1+i+1-i+1-\sqrt{2}+1+\sqrt{2}$ =4 (sum of the roots)

S_4 = $(1+i)(1-i)(1-\sqrt{2})(1+\sqrt{2})$ (product of the roots)

$= (1-i^2)(1-2)$

= -2

Now verify options.

6. To remove the 2nd term of the equation $x^4-8x^3+x^2-x+3=0$ diminished the root of the equation by [**EAMCET-2002**]

1. 1 2. 2 3. 3 4. 4

Ans: 2

Sol. $h=\frac{-a_1}{na_0}=\frac{-(-8)}{4(1)}=2$

7. The maximum possible number of real roots of the equation $x^5-6x^2-4x+5=0$ is

1. 3 2. 4 3. 5 4. 0

Ans: 1

Sol. Let $f(x) = x^5 - 6x^2 - 4x + 5 = 0$, $f(-x) = -x^5 - 6x^2 + 4x + 5 = 0$

Number of positive real roots = Number of changes of signs in f(x)

= 2

No. of negative roots = No. of changes of signs in f(-x)

= 1

$\therefore$ No. of real roots = No. of positive roots + No. of negative roots =

= 2 +1 = 3

8. If α, β, γ are the roots of the equation $x^3+ax^2+bx+c=0$ then $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=$

1. $\frac{a}{c}$ 2. $-\frac{b}{c}$ 3. $\frac{c}{a}$ 4. $\frac{b}{a}$

Ans: 2

Sol. $\alpha^{-1}+\beta^{-1}+\gamma^{-1}=\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} = \frac{S_2}{S_3} = \frac{b}{-c}$$

9. If $\frac{1+\sqrt{3}i}{2}$ is a root of the equation $x^4 - x^3 + x - 1 = 0$ then its real roots are

1. 1,1 2. -1, -1 3. 1, 2 4. 1, -1

Ans: 4

Sol. If $\frac{1+\sqrt{3}i}{2}$ is a roots of the given equation then the other root be roots are $\frac{1-\sqrt{3}i}{2}$

Let the remaining roots be α, β

Now sum of the roots of given equation = S_1 = 1

$$\frac{1+\sqrt{3}\,i}{2} + \frac{1-\sqrt{3}\,i}{2} + \alpha + \beta = 1$$

$$1 + \alpha + \beta = 1$$

$$\alpha + \beta = 0$$

By verification roots are 1,-1

10. If α, β, γ are the roots of $2x^3 - 2x - 1 = 0$ then

1. -1 2. 1 3. 2 4. 3

Ans: 2

Sol. $(\Sigma\alpha\beta)^2 = (S_2)^2$

$$= \left(\frac{-2}{2}\right)^2 = 1$$

11. If α, β, γ are the roots of the equation $x^3 + 4x + 1 = 0$ then $(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} =$

EAMCET - 2003

1) 2 2) 3 3) 4 4) 5

Ans: 3

Sol. $\alpha + \beta + \gamma = 0$

$$(\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} = (-\gamma)^{-1} + (-\alpha)^{-1} + (-\beta)^{-1}$$

$$= -\frac{1}{\gamma} - \frac{1}{\alpha} - \frac{1}{\beta}$$

$$= -\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$$

$$= -\left(\frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right) = -\left(\frac{4}{-1}\right) = 4$$

12. Let $\alpha \neq 0$ and P(x) be a polynomial of degree greater than 2. If P(x) leaves remainders α and $-\alpha$ when divided respectively by $x + \alpha$ and $x - \alpha$ then the remainder when P(x) is divided by $x^2 - \alpha^2$ is

1) 2x 2) -2x 3) x 4) –x

Ans: 4

Sol. Let the remainder be R(x), then

R(x) = p(x)+q and R(a) = -a

Given R(-a) = a pa+q = -a------(2)

- pa + q = a -------(1)

Solving (1) & (2), we get

p = -1, q = 0

$\therefore R(x) = -x$

13. If the sum of two of the roots of $x^3 + px^2 + qx + r = 0$ is zero then pq =

1) -r 2) r 3) 2r 4) -2r

Ans: 2

Sol. Let the roots be α, β, γ

Given $\alpha + \beta = 0$

$\alpha + \beta + \gamma = -p \Rightarrow \gamma = -p$

$\gamma = -p$ is a root of $x^3 + px^2 + qx + r = 0$

$\Rightarrow (-p)^3 + p(-p)^2 + q(-p) + r = 0$

$\therefore pq = r$

14. If the roots of the equation $4x^3 - 12x^2 + 11x + k = 0$ are in A.P. Then K = **[EAMCET-2004]**

1) -3 2) 1 3) 2 4) 3

Ans: 1

Sol. Let the roots be a-d, a, a+d

(a-d) + a + (a+d) = $-\left(\frac{-12}{4}\right)$

3a = 3 $\Rightarrow a = 1$

a = 1 is a root of $4x^3-12x^2+11x+k = 0$

$\Rightarrow 4(1)^3-12(1)^2+11(1)+k=0$

$\Rightarrow$ 3+k = 0 $\therefore$ k = -3

15. α, β, γ are the roots of the equation $x^3 - 10x^2 + 7x + 8 = 0$ Match the following

1) $\alpha + \beta + \gamma$ a) $-\frac{43}{4}$

2) $\alpha^2 + \beta^2 + \gamma^2$ b) $\frac{-7}{8}$

3) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ c) 86

4) $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha} + \frac{\gamma}{\alpha\beta}$ d) 0

e) 10

1) e, c, a, b 2) d, c, a, b 3) e, c, b, a 4) e, b, c, a

Ans: 3

Sol. $x^3 - 10x^2 + 7x + 8 = 0$

Now $\alpha + \beta + \gamma = 10$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= (10)^2 - 2(7)$

$= 86$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{7}{-8}$$

$$\frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha} + \frac{\alpha}{\alpha\beta} = \frac{\alpha^2 + \beta^2 + \gamma^2}{\alpha\beta\gamma} = \frac{86}{-8} = \frac{-43}{4}$$

16. If f(x) is a polynomial of degree n with rational coefficients and 1 + 2i, $2 - \sqrt{3}$ and 5 are three roots of f(x)=0, then the least value of n is

1) 5 2) 4 3) 3 4) 6

Ans: 1

Sol. Since 1+2i, $2 - \sqrt{3}$ and 5 are the some roots of polynomial f(x) of degree n. As we know this conjugate are also the roots of the polynomial is 1-2i, $2 + \sqrt{3}$

$\therefore$ The least value of n is 5.

17. The roots of the equation $x^3 - 3x - 2 = 0$ are **[EAMCET-2005]**

1) -1, -1, 2 2) -1, 1, -2 3) -1, 2, -3 4) -1, -1, -2

Ans 1

Sol. Verify S_1

Here $S_1 = 0$

By verification the roots are -1,-1,2

18. If α, β, γ are the roots of $x^3 + 2x^2 - 3x - 1 = 0$ then $\alpha^{-2} + \beta^{-2} + \gamma^{-2} =$

1) 12 2) 13 3) 14 4) 15

Ans: 2

Sol. $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$

$$= \frac{\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2}{\alpha^2\beta^2\gamma^2}$$

$\alpha\beta\gamma = -2$

$\alpha\beta + \beta\gamma + \gamma\alpha = -3$

$\alpha\beta\gamma = 1$

$(\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$9 = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2(1)(-2)$

$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = 13$

$\alpha^{-2}+\beta^{-2}+\gamma^{2}=\frac{13}{1}=13$

19 The difference between two roots of the equation $x^3-13x^2+15x+189=0$ is 2. Then the roots of the equation are [EAMCET : 2006]

1) -3,5,9 2) -3,-7,-9 3) 3,-5,7 4) -3,7, 9

Ans: 4

Sol. Verify S_1

20. If α,β,γ are the roots of the equation $x^3-6x^2+11x+6=0$ then $\Sigma\alpha^2\beta+\Sigma\alpha\beta^2$ is equal to

1) 80 2) 84 3) 90 4) -84

Ans: 2

Sol. $\Sigma\alpha^2\beta+\Sigma\alpha\beta^2=S_1S_2-3S_3$

= (6) (11) – 3(-6)

= 84

21. If 1, 2, 3 and 4 are the roots of the equation $x^4+ax^3+bx^2+cx+d=0,$ then $a+2b+c=$ **(E-2007)**

1) -25 2) 0 3) 10 4) 24

Ans: 3

Sol. $(x-1)(x-2)(x-3)(x-4) = x^4+ax^3+bx^2+cx+d$

$\Rightarrow\left(x^2-3x+2\right)\left(x^2-7x+12\right)=x^4+ax^3+bx^2+cx+d$

$\Rightarrow x^4-10x^3+35x^2-50x+24=x^4+ax^3+bx^2+cx+d$

Now a = -10, b = 35, c = -50, d = 24

a +2b+c=-10+2(35)-50

= 10

22. If α,β,γ are the roots of $x^3-2x^2+3x-4=0$ then the value of $\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2$ is

1) -7 2) -5 3) -3 4) 0

Ans: 1

Sol. $\alpha+\beta+\gamma=2$, $\alpha\beta+\beta\gamma+\gamma\alpha=3$, $\alpha\beta\gamma=-4$

$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$

$=(3)^2-2(4)(2)$ = -7

EAMCET 2008

23. The cubic equation whose roots are thrice to each of the roots of $x^3-2x^2-4x+1=0$ is

1) $x^3-6x^2+36x+27=0$ 2) $x^3+6x^2+36x+27=0$ 3) $x^3-6x^2-36x+27=0$ 4) $x^3+6x^2-36x+27=0$

Ans: 4

Sol. $x=3\alpha\Rightarrow\frac{x}{3}\Rightarrow f\left(\frac{x}{3}\right)=0$

$\left(\frac{x}{3}\right)^3+2\left(\frac{x}{3}\right)^2-4\left(\frac{x}{3}\right)+1=0$

$\Rightarrow x^3+6x^2-36x+27=0$

24. The sum of fourth powers of the roots of the equation $x^3+x+1=0$ is

1) -2 2) -1 3) 1 4) 2

Ans: 4

Sol. Let roots be α, β, γ we have to find $\alpha^4+\beta^4+\gamma^4$

Let f(x) = x^3+x+1

$f^1(x) = 3x^2+1$

$$\text{Now } -\frac{f^1(x)}{f(x)} = -\frac{(3x^2+1)}{x^3+x+1}$$

	3	0	1	
−1	0	0	3	−1
−1	0	−3	−3	−1
	3	−3	1	2

$$\therefore \ \alpha^4+\beta^4+\gamma^4 = 2$$

25. If α, β, γ are the roots of $x^3+4x+1=0$ then the equation whose roots are $\frac{\alpha^2}{\beta+\gamma}, \frac{\beta^2}{\gamma+\alpha}, \frac{\gamma^2}{\alpha+\beta}$ is

1) $x^3-4x-1=0$ 2) $x^3-4x+1=0$ 3) $x^3+4x-1=0$ 4) $x^3+4x+1=0$

[EAMCET 2009]

Ans: 3

Sol. Let $y = \frac{\alpha^2}{\beta+\gamma} = \frac{\alpha^2}{\alpha} = -\alpha = -x \ [\because \alpha+\beta+\gamma=0]$

$\therefore$ Required equation is $(-x)^3+4(-x)+1=0$

$\Rightarrow x^3+4x-1=0$

26. If $f(x)=2x^4-13x^2+ax+b$ is divisible by x^2-3x+2, then (a,b) =

1) (-a,-2) 2) (6,4) 3) (9,2) 4) (2,9)

Ans: 3

Sol. $x^2-3x+2 = (x-1)(x-2)$

f(1)=0, f(2)=0

2-13+a+b=0 32-52+2a+b=0

a+b=11 2a+b=20

Solving (1) & (2) we get

(a,b) = (9,2)