TANGENT AND NORMALS

PREVIOUS EAMCET BITS

1. The equation to the normal to the curve $y^4 = ax^3$ at (a, a) is **[EAMCET 2008]**
   1) $x + 2y = 3a$ 2) $3x - 4y + a = 0$ 3) $4x + 3y = 7a$ 4) $4x - 3y = 0$
   Ans: 3

Sol. $y^4 = ax^3 \Rightarrow 4y^3 \frac{dy}{dx} = 3ax^2 \Rightarrow \frac{3ax^2}{4y^3} \Rightarrow m = \left(\frac{dy}{dx}\right)_{(a,a)} = \frac{3a^3}{4a^3} = \frac{3}{4}$

Equation of the normal is $y - a = -\frac{4}{3}(x - a) \Rightarrow 4x + 3y = 7a$

2. The length of the substangent at (2, 2) to the curve $x^5 = 2y^4$ is **[EAMCET 2008]**
   1) 5/2 2) 8/5 3) 2/5 4) 5/8
   Ans: 2

Sol. $x^5 = 2y^4 \Rightarrow 5x^4 = 8y^3 \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{5x^4}{8y^3} = \frac{5(2)^4}{8(2)^3} = \frac{5}{4}$

Length of the subtangent $\frac{2}{m} = \frac{2}{5/4} = \frac{8}{5}$

3. The length of tangent, subtangent normal and subnormal for the curve $y = x^2 + x - 1$ at (1, 1) are A, B, C and D respectively, then their increasing order is **[EAMCET 2007]**
   1) B, D, A, C 2) B, A, C, D 3) A, B, C, D 4) B, A, D, C
   Ans:4

Sol. $m = \left(\frac{dy}{dx}\right)_{(1,1)} = 3$

A = length of tangent = $\frac{\sqrt{10}}{3}$

B = length of subtangent = $\frac{1}{3}$

C = length of normal = $\sqrt{10}$

D = length of subnormal = 3

$\therefore$ B < A < D < C

4. If θ is the angle between the curves $xy = 2$ and $x^2 + 4y = 0$, then tanθ = **[EAMCET 2006]**
   1) 1 2) – 1 3) 2 4) 3
   Ans: 4

Sol. $y = \frac{2}{x}$, $y = \frac{-x^2}{4}$

$x^3 = -8$ $(-2, -1)$

$x = -2$

$y = -1$

$m_1 = \frac{dy}{dx} = \frac{-2}{x^2} = \frac{-1}{2}$

$m_2 = \frac{dy}{dx} = \frac{-2x}{4} = \frac{-x}{2} = 1$

$\tan\theta = \left|\frac{m_1 + m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{2} - 1}{1 - \frac{1}{2}}\right| = \left|\frac{\frac{3}{2}}{\frac{1}{2}}\right| = 3$

5. Match the points on the curve $2y^2 = x + 1$ with the slopes of normals at those points and choose the correct answer. **[EAMCET 2004]**

List –I	List – II
I) (7, 2)	a) $-4\sqrt{2}$
II) $\left(0, \frac{1}{\sqrt{2}}\right)$	b) – 8
III) (1, –1)	c) 4
IV) $\left(3, \sqrt{2}\right)$	d) 0
	e) $-2\sqrt{2}$

1) b, d, c, a 2) b, e, c, a 3) b, c, e, a 4) b, e, a, c

Ans: 2

Sol. $2y^2 = x + 1 \Rightarrow \frac{-dx}{dy} = -4y = \text{slope of the normal}$

$-\frac{dx}{dy} \text{at} (7, 2) = -8$

$-\frac{dx}{dy} \text{at} \left(0, \frac{1}{\sqrt{2}}\right) = -2\sqrt{2}$

$-\frac{dx}{dy} \text{at} (1, -1) = 4$;

$-\frac{dx}{dy} \text{at} \left(3, \sqrt{2}\right) = -4\sqrt{2}$

6. The angle between the curves $y = \sin x$ and $y = \cos x$ is **[EAMCET 2003]**

1) $\tan^{-1}\left(2\sqrt{2}\right)$ 2) $\tan^{-1}\left(3\sqrt{2}\right)$ 3) $\tan^{-1}\left(3\sqrt{3}\right)$ 4) $\tan^{-1}\left(5\sqrt{2}\right)$

Ans: 1

Sol. $y = \sin x; y = \cos x \Rightarrow x = \frac{\pi}{4}$

$m_1 = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$m_2 = -\sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

$\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right) = \tan^{-1}\left(2\sqrt{2}\right)$

7. The two curves $x = y^2$, $xy = a^3$ cut orthogonally at a point, then $a^2 =$ **[EAMCET 2002]**

1) 1/3 2) 1/2 3) 2 4) 3

Ans: 2

Sol. $x = y^2, xy = a^3$

$2ym_1 = 1\ m_2 = \frac{-a^3}{x^2} \Rightarrow m_1 = \frac{1}{2y}$

$m_1 m_2 = -1 \Rightarrow \frac{1}{2y}.\frac{a^3}{x^2} = +1$

$\frac{xy}{2x^2y} = 1 \Rightarrow x = \frac{1}{2}$

$a^6 = x^2y^2 = x^3 = \frac{1}{8} \Rightarrow \left(a^2\right)^3 = \left(\frac{1}{2}\right)^3 \ \therefore\ a^2 = \frac{1}{2}$

8. The equation of the tangent to the curve $6y = 7 - x^3$ at (1, 1) is **[EAMCET 2001]**

1) 2x + y = 3 2) x + 2y = 3 3) x + y = - 1 4) x + y + 2 = 0

Ans: 2

Sol. $6y = 7 - x^3 \Rightarrow \frac{dy}{dx} \text{at } (1,1) = -\frac{1}{2}$

Equation of the tangent is $y - 1 = \frac{-1}{2}(x - 1)$

$\Rightarrow x + 2y - 3 = 0$

9. The angle between the curves $y^2 = 4x$, $x^2 = 4y$ at $(4,4)$ is **[EAMCET 2000]**

1) $\tan^{-1}\left(\frac{1}{2}\right)$ 2) $\tan^{-1}\left(\frac{3}{4}\right)$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

Ans: 2

Sol. $y^2 = 4x \Rightarrow \frac{dy}{dx} \text{at } (4,4) = \frac{1}{2} = m_1$

$x^2 = 4y \Rightarrow \frac{dy}{dx} \text{at } (4,4) = 2 = m_2$

$\operatorname{Tan}\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$

10. Area of the triangle formed by the normal to the curve $x = e^{\sin y}$ at $(1,0)$ with the coordinate axes is **[EAMCET 2000]**

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) $\frac{3}{4}$ 4) 1

Ans: 2

Sol. $x = e^{\sin y} \Rightarrow \log x = \sin y \Rightarrow \frac{dy}{dx} = \frac{1}{x \cos y}$

$\frac{dy}{dx} \text{at } (1,0) = 1 =$ slope of the tangent

$\therefore$ Equation of the normal is x + y – 1 = 0

$\therefore$ Area of the triangle with normal and coordinate axes = 1/ 2

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