SUCCESSIVE – DIFFERENTIAL

PREVIOUS EAMCET BITS

1. $y = e^{a\sin^{-1}x} \Rightarrow (1-x^2)y_{n+2} - (2n+1)xy_{n+1} =$ **[EAMCET 2009]**

1) $(n^2+n^2)y_n$ 2) $(n^2-a^2)y_n$ 3) $(n^2+a^2)y_n$ 4) $-(n^2-a^2)y_n$

Ans: 3

Sol: $y = e^{a\sin^{-1}x}, \Rightarrow y_1 = y \;x \dfrac{a}{\sqrt{1-x^2}}$

$\Rightarrow (1-x^2)y_1^2 = a^2y^2$

$\Rightarrow 2(1-x^2)y_1y_2 - 2xy_1^2 = 2a^2yy_1$

$(1-x^2)y_2 - xy_1 = a^2y$ ………….(1)

Diff. (1) 'n' times using Leibnitz theorem $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} = (n^2+a^2)y_n$

2. If $y = \sin(\log_c x)$ then $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} =$ **[EAMCET 2008]**

1) $\sin(\log_e x)$ 2) $\cos(\log_e x)$ 3) y^2 4) $-y$

Ans: 4

Sol: $y = \sin(\log x) \Rightarrow \dfrac{dy}{dx} = \cos(\log x)\dfrac{1}{x} \Rightarrow x\dfrac{dy}{dx} = \cos(\log x)$

$\Rightarrow x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = -\sin(\log x)\dfrac{1}{x}$

$\Rightarrow x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = -y$

3. $x = \cos\theta, y = \sin 5\theta \Rightarrow (1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} =$ **[EAMCET 2007]**

1) $-5y$ 2) 5y 3) 25 y 4) –25 y

Ans: 4

Sol: $\dfrac{dy}{dx} = \dfrac{-5\cos 5\theta}{\sin\theta} = \dfrac{-5\sqrt{1-\sin^2 5\theta}}{\sqrt{1-\cos^2\theta}}$

$= -5\sqrt{\dfrac{1-y^2}{1-x^2}} = y_1$

$(1-x^2)y_1^2 = 25(1-y^2) \Rightarrow (1-x^2)y_2 - xy_1 = -25y$

4. $f(x) = e^x \sin x \Rightarrow f^{(6)}(x) =$ **[EAMCET 2006]**

1) $e^{6x}\sin 6x$ 2) $-8e^x \cos x$ 3) $8e^x \sin x$ 4) $8e^x \cos x$

Ans: 2

Sol: $f(x) = e^{ax}\sin bx$

$f^n(x) = \left(a^2+b^2\right)^{n/2}.e^{ax}\sin\left(bx + n\tan^{-1} b/a\right)$

$a = 1, b = 1, n = 6$

$f^6(x) = \left(\sqrt{(1+1)}\right)^6 e^x \sin\left(x + 6\tan^{-1}(1)\right)$

$= 8e^x \sin\left(\frac{3\pi}{2} + x\right) = -8e^x \cos x$

5. $y = \sin^{-1} x \Rightarrow \left(1-x^2\right)\frac{d^2y}{dx^2} =$ **[EAMCET 2004]**

1) $-x\frac{dy}{dx}$ 2) 0 3) $x\frac{dy}{dx}$ 4) $x\left(\frac{dy}{dx}\right)^2$

Ans: 3

Sol: $y = \sin^{-1} x \Rightarrow y_1 = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \left(1-x^2\right)y_1^2 = 1$

$\Rightarrow \left(1-x^2\right)2y_1y_2 - 2xy_1^2 = 0$

$\therefore \left(1-x^2\right)y_2 = xy_1$

6. If $I_n = \frac{d^n}{dx^n}\left(x^n \log x\right)$, then $I_n - nI_{n-1} = ...$ **[EAMCET 2003]**

1) n 2) n – 1 3) n! 4) (n –1)!

Ans: 4

Sol: $I_n = \frac{d^n}{dx^n}\left(x^n \log x\right)$

$y = x^n \log x \Rightarrow y_1 = x^n\left(\frac{1}{x}\right) + nx^{n-1}\log x$

$\left(y_1\right)_{n-1} = nI_{n-1} + (n-1)!$

$\Rightarrow I_n - nI_{n-1} = (n-1)!$

7. If $y = ae^x + be^{-x} + c$, where a, b, c are parameters, then $y''' =$ **[EAMCET 2002]**

1) y 2) y' 3) 0 4) y''

Ans: 2

Sol. $y = ae^{x} + be^{-x} + c$

$y' = ae^{x} - be^{-x};$

$y'' = ae^{x} + be^{-x}$

$y''' = ae^{x} - be^{-x}$

$y''' = y'$

8. If $y = a\cos(\log x) + b\sin(\log x)$, where a, b are parameters, then $x^2y'' + xy' =$ **[EAMCET 2002]**

1) y 2) – y 3) 2y 4) –2y

Ans: 2

Sol: $y = a\cos(\log x) + b\sin(\log x)$

$xy' = -a\sin(\log x) + b\cos(\log x)$

$xy'' + y' = \dfrac{-a\cos(\log x) - b\sin(\log x)}{x}$

$\Rightarrow x^2y'' + xy' = -y$

9. If y_k is the k^{th} derivative of y with respect to x, y = cos(sinx) then $y_1 \sin x + y_2 \cos x =$ **[EAMCET 2001]**

1) $y\sin^3 x$ 2) $-y\sin^3 x$ 3) $y\cos^3 x$ 4) $-y\cos^3 x$

Ans: 4

Sol: Given y = cos(sinx)

$\Rightarrow y_1 = -\sin(\sin x).\cos x$

$y_2 = \sin(\sin x).\sin x - \cos^2 x.\cos(\sin x)$

$\therefore y_1 \sin x + y_2 \cos x = -\sin(\sin x).\sin x \cos x$

$+\sin(\sin x)\sin x.\cos x - \cos^3 x.\cos(\sin x) = -y\cos^3 x$

10. $\dfrac{d^n}{dx^n}(e^x \sin x) =$ **[EAMCET 2000]**

1) $2^{n/2}.e^x \cos(x + n\pi/4)$ 2) $2^{n/2}.e^x \cos(x - n\pi/4)$

3) $2^{n/2}.e^x \sin(x + n\pi/4)$ 4) $2^{n/2}.e^x \sin(x - n\pi/4)$

Ans: 3

Sol: $y = e^{ax}\sin(bx) \Rightarrow y_n = \left(\sqrt{a^2 + b^2}\right)^n .e^{ax} \sin\left(bx + n\tan^{-1}\dfrac{b}{a}\right)$

where a = 1, b = 1

$$y = e^x \sin x \quad \Rightarrow y_n = 2^{n/2} e^x \sin\left(x + \frac{n\pi}{4}\right)$$

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