9. RANDOM VARIABLE AND DISTRIBUTIONS

PREVIOUS EAMCET BITS

1. If m and σ^2 are the mean and variance of the random variable X, whose distribution is given by

X = x :	0	1	2	3
P(X = x)	1/3	1/2	0	1/6

   Then **[EAMCET 2009]**

   1) $m = \sigma^2 = 2$ 2) $m = 1, \sigma^2 = 2$ 3) $m = \sigma^2 = 1$ 4) $m = 2, \sigma^2 = 1$

   Ans:

Sol : $m = 0 + \dfrac{1}{2} + 0 + \dfrac{3}{6} = 1$

   $\sigma^2 = \displaystyle\sum_n x_n^2 \cdot p_n - m^2 = 1$

2. If X is a binomial variate with the range $\{0,1,2,3,4,5,6\}$ and $P(X = 2) = 4\,P(X = 4)$, then the parameter p of X is **[EAMCET 2009]**

   1) $\dfrac{1}{3}$ 2) $\dfrac{1}{2}$ 3) $\dfrac{2}{3}$ 4) $\dfrac{3}{4}$

   Ans: 1

Sol: Given $n = 6, P(X = 2) = 4P(X = 4)$

   $\Rightarrow 6_{c_2} p^2 q^4 = 4 \times 6_{c_4} p^4 q^2$

   $\Rightarrow q = 2p \Rightarrow 1 - p = 2p$

   $\Rightarrow p = 1/3$

3. The distribution of a random variable X is given below

X = x	-2	-1	0	1	2	3
P(X = x)	$\dfrac{1}{10}$	K	$\dfrac{1}{5}$	2k	$\dfrac{3}{10}$	k

   the value of k is **[EAMCET 2008]**

   1) $\dfrac{3}{10}$ 2) $\dfrac{2}{10}$ 3) $\dfrac{3}{10}$ 4) $\dfrac{7}{10}$

   Ans:

Sol: $\displaystyle\sum P(X = x) = 1 \Rightarrow \dfrac{1}{10} + k + \dfrac{1}{5} + 2k + \dfrac{3}{10} + k = 1$

   $\Rightarrow 4k = \dfrac{4}{10} \Rightarrow k = \dfrac{1}{10}$

4. If X is a Poisson variate such that $P(X=1)P(X=2)$, then $P(X=4)$ **[EAMCET 2008]**

    1) $\dfrac{1}{2e^2}$
    2) $\dfrac{1}{3e^2}$
    3) $\dfrac{2}{3e^2}$
    4) $\dfrac{1}{e^2}$

    Ans: 3

Sol: $P(X=1)=P(X=2)\Rightarrow \dfrac{e^{-\lambda}\lambda}{1!}=\dfrac{e^{-\lambda}\lambda^2}{2!}$

$\Rightarrow \lambda=2. P(X=4)=\dfrac{e^{-\lambda}\lambda^4}{4!}=\dfrac{e^{-2}2^4}{4!}=\dfrac{2}{3e^2}$

5. The mean and standard deviation of a binomial variate X are 4 and $\sqrt{3}$ respectively. Then $P(X\geq 1)$ is equal to **[EAMCET 2007]**

    1) $1-\left(\dfrac{1}{4}\right)^{16}$
    2) $1-\left(\dfrac{3}{4}\right)^{16}$
    3) $1-\left(\dfrac{2}{3}\right)^{16}$
    4) $1-\left(\dfrac{1}{3}\right)^{16}$

    Ans: 2

Sol: Mean = np = 4, variance nqp = 3

On solving, we get $q=\dfrac{3}{4}, n=16, p=\dfrac{1}{4}$

Now $p(X\geq 1)=1-p(X=0)=1-{}^n C_0 p^0 q^{n-0}=1-\left(\dfrac{3}{4}\right)^{16}$

6. The probability distribution of a random variable X is given by

X = x	0	1	2	3	4
P(X = x)	0.4	0.3	0.1	0.1	0.1

    The variance of X is **[EAMCET 2007]**

    1) 1.76
    2) 2.45
    3) 3.2
    4) 4.8

    Ans: 1

Sol: Given

X = x	P(X = x)	xP(X = x)	x^2P
0	0.4	0	0
1	0.3	0.3	0.3
2	0.1	0.2	0.4
3	0.1	0.3	0.9
4	0.1	0.4	1.6

Mean $\overline{x}=\displaystyle\sum_{i=0}^{4} p_i x_i =1.2$

Variance $= \sum_{i=0}^{4} p_i x_i^2 - \overline{x}^2 = 3.20 - 1.44 = 1.76$

7. In a book of 500 pages, it is found that there are 250 typing errors. Assume that Poisson law holds for the number of errors per page. Then, the probability that a random sample of 2 pages will contain no error, is **[EAMCET 2006]**

   1) $e^{-0.3}$ 2) $e^{-0.5}$ 3) e^{-1} 4) e^{-2}

   Ans: 3

Sol: Here number of errors per page $= \dfrac{250}{500} = \dfrac{1}{2}$

   and n = 2

   $\therefore \lambda = np = 2 \times \dfrac{1}{2} = 1$ and probability of no error $P(X = 0) = \dfrac{e^{-1} \times (1)^0}{0!} = e^{-1}$

8. If the range of a random variable X is {0, 1, 2, 3, 4, ...} with $P(X = k) = \dfrac{(k+1)a}{3^k}$ for $k \geq 0$, then a is equal to **[EAMCET 2005]**

   1) $\dfrac{2}{3}$ 2) $\dfrac{4}{9}$ 3) $\dfrac{8}{27}$ 4) $\dfrac{16}{81}$

   Ans: 2

Sol: Given that $P(X = k) = \dfrac{(k+1)}{3^k}$ for $x \in \{0, 1, 2, ...\infty\}$

   As we know that $P(0) + P(1) + P(2) + ...\infty = 1$

   $\Rightarrow a + \dfrac{2a}{3} + \dfrac{3a}{3^2} + ...\infty = 1 .....(i)$

   $S = a \left(1 + \dfrac{2}{3} + \dfrac{3}{3^2} + \dfrac{4}{3^3} + ...\infty \right) \Rightarrow \begin{array}{l} \dfrac{1}{3}S = a \left(\dfrac{1}{3} + \dfrac{2}{3^2} + \dfrac{3}{3^3} + ...\infty \right) \\ \hline S - \dfrac{1}{3}S = a \left(1 + \dfrac{1}{3} + \dfrac{1}{3^2} + \dfrac{1}{3^3} + ...\infty \right) \end{array}$

   $\Rightarrow \dfrac{2}{3}S = a \left(\dfrac{1}{1 - \dfrac{1}{3}} \right) \Rightarrow \dfrac{2}{3}S = \dfrac{3a}{2} \Rightarrow S = \dfrac{9a}{4}$

   from equation (i) $\Rightarrow \dfrac{9a}{4} = 1 \Rightarrow a = \dfrac{4}{9}$

9 For a binomial variate X with n = 6, if P(X = 2) = 9P(X = 4), then its variance is **[EAMCET 2005]**

   1) $\dfrac{8}{9}$ 2) $\dfrac{1}{4}$ 3) $\dfrac{9}{8}$ 4) 4

Ans: 3

Sol: Given that n = 6 and $P(X = 2) = 9P(X = 4)$

$$\Rightarrow {}^6C_2 p^2 q^4 = 9.{}^6C_4 p^4 q^2$$

$$\Rightarrow 9p^2 = q^2 \Rightarrow p = \frac{1}{3}q$$

$\because$ we know that p + q = 1

$$\Rightarrow \frac{q}{3} + q = 1 \Rightarrow q = \frac{3}{4} \text{ and } p = \frac{1}{4}$$

$\therefore$ variance = npq = $6.\frac{1}{4}.\frac{3}{4} = \frac{9}{8}$

10. A person who tosses an unbiased coin gains two points for turning up a head and loses one point for a tail. If three coins are tossed and the total score X is observed, then the range of x is

    [EAMCET 2004]

    1) $\{0.3.6\}$ 　　　　2) $\{-3,0,3\}$ 　　　　3) $\{-3,0,3,6\}$ 　　　　4) $\{-3,3,6\}$

    Ans: 3

Sol: Since it is given that for tossing a coin, if head will come down it will give two point and for tail comes down it loose one point.

    There are four case arise : Case(i) If all three tails comes out, then his points = – 1 –1 –1 = -3

    Case (ii) If two tails and one head comes out, then his points = – 1 –1 + 2 = 0

    Case (iii) If one tail and two heads comes out, then this points = –1 + 2 + 2 =3

    Case (iv) If all three heads comes out, then his points = 2 + 2 + 2 = 6

    $\therefore$ Range = {– 3, 0, 3, 6}

11. If X is a Poisson variate with P(X = 0) = 0.8, then the variance of X is 　　　　**[EAMCET 2004]**

    1) $\log_e 20$ 　　　　2) $\log_{10} 20$ 　　　　3) $\log_e 5/4$ 　　　　4) 0

    Ans: 3

Sol: Poisson distribution $P(X) = \frac{e^{-m} m^x}{x!}$

    $$\therefore P(X = 0) = \frac{e^{-m} 1}{1} \Rightarrow 0.8 = e^{-m} \Rightarrow -m = \log_e 0.8$$

    $$\Rightarrow m = \log_e \frac{10}{8} = \log_e \frac{5}{4}$$

    As we know in a Poisson distribution variance = m

    Variance = $\log_e \frac{5}{4}$

12. For a Poisson variate X, if $P(X = 2) = 3P(X = 3)$, then the mean of X is **[EAMCET 2003]**

    1) 1   2) $\dfrac{1}{2}$ 3) $\dfrac{1}{3}$ 4) $\dfrac{1}{4}$

    Ans: 1

Sol: We known that $P(X = k) = \dfrac{\lambda^k e^{\lambda}}{k!}$

    Since, $P(X = 2) = 3P(X = 3)$

    $\Rightarrow \dfrac{\lambda^2 e^{\lambda}}{2!} = 3.\dfrac{\lambda^3 e^{\lambda}}{3!} \Rightarrow \lambda = 1$

    $\therefore$ Mean of Poisson distribution $= \lambda \Rightarrow 1$

13. A random variable X takes the values 0, 1 , 2, 3 and its mean is 13. If $P(X = 3) = 2P(X = 1)$ an $P(X = 2) = 0.3$, then $P(X = 0)$ is equal to **[EAMCET 2003]**

    1) 0.1   2) 0.2   3) 0.3   4) 0.4

    Ans: 4

Sol: Given that $\text{Mean} = \sum X_k P(X = k) = 1.3$

    $X_0 P(X = 0) + X_1 P(X = 1) + X_2 P(X = 2) + X_3 P(X = 3) = 1.3$

    $\Rightarrow 0.P(X = 0) + 1.P(X = 1) + 2.P(X = 2) + 3.P(X = 3) = 1.3$

    $\Rightarrow P(X = 1) + 2(0.3) + 3.2P(X = 1) = 1.3$

    $\Rightarrow 7P(X = 1) = 0.7 \Rightarrow P(X = 1) = 0.1$

    Now, $P(X = 3) 2P(X = 1) = 2(0.1) = 0.2$

    Also, $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1$

    $\Rightarrow P(X = 0) + 0.1 + 0.3 + 0.2 = 1 \Rightarrow P(X = 0) = 1 - 0.6 = 0.4$

14. In a binomial distribution the probability of getting success is 1/4 and the standard deviation is 3. Then its mean is **[EAMCET 2002]**

    1) 6   2) 8   3) 10   4) 12

    Ans:4

Sol: Given that $P = \dfrac{1}{4}$ and $q = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

    $S.D = 3 \Rightarrow \sqrt{npq} = 3 \Rightarrow npq = 9 \Rightarrow n.\dfrac{1}{4}.\dfrac{3}{4} = 9 = n = 48$

    $\text{Mean} = np = 48 \times \dfrac{1}{4} = 12$

15. If the mean of a Poisson distribution is 1/2 then the ratio of $P(X = 3)$ to $P(X = 2)$ is

    [EAMCET 2002]

    1) 1 : 2   2) 1 : 4   3) 1 : 6   4) 1 : 8

    Ans: 3

Sol: Given that $\lambda = \dfrac{1}{2}$, Now $P(X = n) = \dfrac{\lambda^n}{n!} e^\lambda$

$$\therefore P(X = 3) = \dfrac{\left(\dfrac{1}{2}\right)^3}{3!} e^{1/2} \text{ and } P(X = 2) = \dfrac{\left(\dfrac{1}{2}\right)^2}{2!} e^{1/2}$$

$$\frac{P(X = 3)}{P(X = 2)} = \dfrac{\dfrac{\left(\dfrac{1}{2}\right)^3 e^{1/2}}{3!}}{\dfrac{\left(\dfrac{1}{2}\right)^2 e^{1/2}}{2!}} = \dfrac{\left(\dfrac{1}{2}\right)^3 e^{1/2} 2!}{\left(\dfrac{1}{2}\right)^2 e^{1/2} 3!} = \dfrac{\dfrac{1}{2}}{3} = \dfrac{1}{6}$$

16. A random variable X takes the values 0, 1 and 2. If $P(X = 1) = P(X = 2)$ and $P(X = 0) = 0.4$, then the mean of the random variable X is **[EAMCET 2002]**

    1) 0.2   2) 0.7   3) 0.5   4) 0.9

    Ans: 4

Sol: We have $P(X = 1) = P(X = 2)$ ………..(i)

$$\frac{\lambda^1}{1!} e^\lambda = \frac{\lambda^2}{2!} e^\lambda \Rightarrow \lambda = 2$$

also $P(X = 0) + P(X = 1) + P(X = 2) = 1$

$$\Rightarrow 0.4 + P(X = 1) + P(X = 2) = 1$$

$$\Rightarrow P(X = 1) + P(X = 2) = 0.6 = \frac{6}{10} = \frac{3}{5}$$

Also, $P(X = 1) + P(X = 1) = \dfrac{3}{5}$ [from (i)]

$$\Rightarrow P(X = 1) = \frac{3}{10} \Rightarrow P(X = 1) = P(X = 2) = \frac{3}{10}$$

Mean $X_0 P(X = 0) + X_1 P(X = 1) + X_2 P(X = 2)$

$$= 0 + 1.\frac{3}{10} + 2.\frac{3}{10} = \frac{9}{10} = 0.9$$

17. Find the binomial probability distribution whose mean is 3 and variance is **[EAMCET 2001]**

    1) $\left(\dfrac{2}{3} + \dfrac{1}{3}\right)^9$ 2) $\left(\dfrac{5}{3} + \dfrac{2}{3}\right)^9$ 3) $\left(\dfrac{3}{3} + \dfrac{1}{2}\right)^9$ 4) None of these

Ans: 1

Sol: We have $np = 3, npq = 2 \Rightarrow q = \dfrac{2}{3}$

$\therefore \ p = 1 - q = 1 - \dfrac{2}{3} = \dfrac{1}{3}, n = 9$

Hence, the binomial distribution is $(q + p)^n = \left(\dfrac{2}{3} + \dfrac{1}{3}\right)^9$

18. For a binomial variate X, if $n = 4$ and $P(X = 4) = 6P(X = 2)$, then the value of p is

[EAMCET 2001]

1) $\dfrac{3}{7}$ 2) $\dfrac{4}{7}$ 3) $\dfrac{6}{7}$ 4) $\dfrac{5}{7}$

Ans: 3

Sol: We have, $P(x = 4) = 6P(x = 2)$

$\Rightarrow ^4C_4 p^4 q^0 = 6 \cdot {}^4C_2 p^2 q^2$

$\Rightarrow p^2 = 36 q^2$

$\Rightarrow p = 6q = 6(1 - p) \Rightarrow p = \dfrac{6}{7}$

19. For all values of a and b the line $(a + 2b)x + (a - b)y + (a + 5b) = 0$ passes through the point

[EAMCET 2001]

1) $(-1, 2)$ 2) $(2, -1)$ 3) $(-2, 1)$ 4) $(1, -2)$

Ans: 3

Sol: Let the line passes through a point whose co-ordinates are (–2 , 1) then

$(a + 2b)(-2) + (a - b)(1) + a + 5b$

$= -2a - 4b + a - b + a + 5b$

$= -2a - 5b + 2a + 5b = 0$

20. The probability distribution of a random variable X is given below, then k is equal to

N	1	2	3	4
P(X = n)	2k	4k	3k	K

[EAMCET 2000]

1) 0.1 2) 0.2 3) 0.3 4) 0.4

Ans: 1

Sol: We have, $P(X = 1) + P(x = 2) + P(X = 3) + P(X = 4) = 1$

$\Rightarrow 2k + 4k + 3k + k = 1 \ \Rightarrow k = \dfrac{1}{10} = 0.1$