

## QUADRATIC EXPRESSIONS

### PREVIOUS EAMCET BITS

1. The roots of  $(x-a)(x-a-1)+(x-a-1)(x-a-2)+(x-a)(x-a-2)=0$ ,  $a \in \mathbb{R}$  always :

- 1) equal            2) imaginary            3) real and distinct            4) rational and equal

Ans: 3

[EAMCET 2009]

Sol : Put  $a = 0$

$$x(x-1)+(x-1)(x-2)+(x)(x-2)=0$$

$$3x^2 - 6x + 2 = 0 \Rightarrow 3x^2 - 6x + 2 = 0$$

$$\Delta = (6)^2 - 4(3)(2)$$

$$\Delta \neq 0$$

$\therefore$  Roots are real and distinct.

2. Let  $f(x) = x^2 + ax + b$ , where  $a, b \in \mathbb{R}$ . If  $f(x) = 0$  has all its roots imaginary, then the roots of

$$f(x) + f'(x) + f''(x) = 0 \text{ are :}$$

[EAMCET 2009]

- 1) real and distinct            2) imaginary            3) equal            4) rational and equal

Ans: 2

Sol. Given roots of  $f(x)$  are imaginary  $\Rightarrow \Delta < 0$   $a^2 - 4b < 0$

$$f'(x) = 2x + a$$

$$f''(x) = 2$$

$$f(x) + f'(x) + f''(x) = 0$$

$$x^2 + ax + b + 2x + a + 2 = 0$$

$$\Rightarrow x^2 + (a+2)x + (a+b+2) = 0$$

$$\therefore \Delta = (a+2)^2 - 4(1)(a+b+2)$$

$$= a^2 + 4a + 4 - 4a - 4b - 8$$

$$= a^2 - 4b - 4 < 0$$

$\therefore$  Roots are imaginary.

3. Let  $\alpha$  and  $\beta$  be the roots of the quadratic equations  $ax^2 + bx + c = 0$  observe the lists given below.

[EAMCET 2008]

LIST - I

LIST - II

i)  $\alpha = \beta \Rightarrow$

ii)  $\alpha = 2\beta \Rightarrow$

iii)  $\alpha = 3\beta \Rightarrow$

iv)  $\alpha = \beta^2 \Rightarrow$

A)  $(ac^2)^{1/3} + (a^2c)^{1/3} + b = 0$

B)  $2b^2 = 9ac$

C)  $b^2 = 6ac$

D)  $3b^2 = 16ac$

E)  $b^2 = 4ac$

F)  $(ac^2)^{1/3} + (a^2c)^{1/3} = b$

The correct match to List – I from List – II is

- |      |    |     |    |      |    |     |    |      |    |     |    |      |    |     |    |
|------|----|-----|----|------|----|-----|----|------|----|-----|----|------|----|-----|----|
| i    | ii | iii | iv | i    | ii | iii | iv | i    | ii | iii | iv | i    | ii | iii | iv |
| 1) E | B  | D   | F  | 2) E | B  | A   | D  | 3) E | D  | B   | F  | 4) E | B  | D   | A  |

Ans: 4

Sol : If the roots are in the ratio  $m : n$  then  $\frac{b^2}{ac} = \frac{(m+n)^2}{mn}$

i)  $\alpha = \beta \Rightarrow \frac{\alpha}{\beta} = \frac{1}{1} \Rightarrow \frac{b^2}{ac} = \frac{(1+1)^2}{1 \times 1} \Rightarrow b^2 = 4ac$

ii)  $\alpha = 2\beta \Rightarrow \frac{\alpha}{\beta} = \frac{2}{1} \Rightarrow \frac{b^2}{ac} = \frac{(2+1)^2}{2 \times 1} \Rightarrow 2b^2 = 9ac$

iii)  $\alpha = 3\beta \Rightarrow \frac{\alpha}{\beta} = \frac{3}{1} \Rightarrow \frac{b^2}{ac} = \frac{(3+1)^2}{3 \times 1} \Rightarrow 3b^2 = 16ac$

iv)  $\alpha = \beta^2 \Rightarrow \beta + \beta^2 = -b/a, \beta^3 = \frac{c}{a}$

$$\Rightarrow \left(\frac{c}{a}\right)^{1/3} + \left(\frac{c}{a}\right)^{2/3} = -b/a$$

$$\Rightarrow (a^2c)^{1/3} + (ac^2)^{1/3} = -b$$

4. If  $\alpha + \beta = -2$  and  $\alpha^3 + \beta^3 = -56$  then the quadratic equation whose roots are  $\alpha$  and  $\beta$  is

- 1)  $x^2 + 2x - 16 = 0$       2)  $x^2 + 2x - 15 = 0$       3)  $x^2 + 2x - 12 = 0$       4)  $x^2 + 2x - 8 = 0$

[EAMCET 2008]

Ans: 4

Sol :  $\alpha^3 + \beta^3 = -56$

$$(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -56$$

$$(-2)^3 + 3\alpha\beta(-2) = -56$$

$$-8 + 6\alpha\beta = -56$$

$$\alpha\beta = -8$$

$\therefore$  Required equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 + 2x - 8 = 0$$

5. If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$  and  $px^2 + qx + r = 0$  has roots

$$\frac{1-\alpha}{\alpha} \text{ and } \frac{1-\beta}{\beta} \text{ then } r =$$

[EAMCET 2007]

1)  $a + 2b$

2)  $a + b + c$

3)  $ab + bc + ca$

4)  $abc$

Ans: 2

Sol. The equation having roots  $\frac{1}{\alpha}, \frac{1}{\beta}$  is  $cx^2 + bx + a = 0$ .

The equation having roots  $\frac{1}{\alpha} - 1, \frac{1}{\alpha} - 1$  is

$$c(x+1)^2 + b(x+1) + a = 0$$

$$\Rightarrow cx^2 + (2c+b)x + (c+b+a) = 0$$

$$px^2 + qx + r = 0$$

$$\therefore r = a + b + c$$

6. The set of values of  $x$  for which the inequalities  $x^2 - 3x - 10 < 0, 10x - x^2 - 16 > 0$  hold simultaneously is [EAMCET 2007]

- 1)  $(-2, 5)$                       2)  $(2, 8)$                       3)  $(-2, 8)$                       4)  $(2, 5)$

Ans: 4

Sol.  $x^2 - 3x - 10 < 0 \Rightarrow (x-5)(x+2) > 0$

$$\Rightarrow -2 < x < 5 \text{ (or) } x > 5$$

$$10x - x^2 - 16 > 0 \Rightarrow x^2 - 10x + 16 < 0$$

$$\Rightarrow (x-2)(x-8) < 0$$

$$\Rightarrow 2 < x < 8$$

$\therefore$  common set =  $(2, 5)$

7. If  $\sqrt{9x^2 + 6x + 1} < (2-x)$  then [EAMCET 2006]

- 1)  $x \in \left(-\frac{3}{2}, \frac{1}{4}\right)$     2)  $x \in \left(\frac{-3}{2}, \frac{1}{4}\right]$     3)  $x \in \left[\frac{-3}{2}, \frac{1}{4}\right)$     4)  $x < 1/4$

Ans: 1

Sol:  $\sqrt{9x^2 + 6x + 1} < (2-x)$

$$\Rightarrow \sqrt{(3x+1)^2} < (2-x)$$

Taking +ve sign

$$3x+1 < (2-x)$$

$$\Rightarrow x < 1/4$$

Taking -ve sign

$$-3x-1 < 2-x$$

$$x > \frac{-3}{2}$$

$$x \in \left(\frac{-3}{2}, \frac{1}{4}\right)$$

8. If  $x$  is real, then the minimum value of  $\frac{x^2 - x + 1}{x^2 + x + 1}$  is [EAMCET 2005]

- 1)  $1/3$                       2)  $3$                       3)  $1/2$                       4)  $1$

Ans: 1

Sol. Let  $y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$$\Rightarrow (y-1)x^2 + (y+1)x + (y-1) = 1$$

$$x \text{ is real } \Rightarrow (y+1)^2 - 4(y-1)^2 \geq 0$$

$$\Rightarrow 3y^2 - 10y + 3 \leq 0$$

$$\Rightarrow (3y-1)(y-3) \leq 0$$

$$\Rightarrow \frac{1}{3} \leq y \leq 3$$

$\therefore$  Minimum value = 1/3

9.  $E_1: a + b + c = 0$  if 1 is a root of  $ax^2 + bx + c = 0$

$E_2: b^2 - a^2 = 2ac$  if  $\sin \theta$  and  $\cos \theta$  are the roots of  $ax^2 + bx + c = 0$

[EAMCET 2005]

1)  $E_1$  is true,  $E_2$  is true

2)  $E_1$  is true,  $E_2$  is false

3)  $E_1$  is false,  $E_2$  is true

4)  $E_1$  is false,  $E_2$  is false

Ans: 1

Sol.  $E_1: 1$  is a root of  $ax^2 + bx + c = 0$

$$\Rightarrow a(1)^2 + b(1) + c = 0$$

$$\Rightarrow a + b + c = 0$$

$E_2: \sin \theta + \cos \theta = -\frac{b}{a}, \sin \theta \cos \theta = c/a$

$$(\sin \theta + \cos \theta)^2 = \left(-\frac{b}{a}\right)^2$$

$$1 + 2\sin \theta \cos \theta = \frac{b^2}{a^2} \Rightarrow 1 + 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2}$$

$$\Rightarrow b^2 - a^2 = 2ac$$

$\therefore E_1$  is true,  $E_2$  is true.

10. The set of all solutions of the inequation  $x^2 - 2x + 5 \leq 0$  is

[EAMCET 2004]

1)  $R - (-\infty, -5)$

2)  $R - (5, \infty)$

3)  $\phi$

4)  $R - (-\infty, -4)$

Ans: 3

Sol.  $x^2 - 2x + 5 = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i$

$$\Rightarrow x^2 - 2x + 5 > 0 \forall x \in \mathbf{R}$$

$\therefore x^2 - 2x + 5 \leq 0$  has no real solution.

11. If  $x-2$  is a common factor of the expressions  $x^2 + ax + b$  and  $x^2 + cx + d$ , then  $\frac{b-d}{c-a} =$

[EAMCET 2004]

1) -2

2) -1

3) 1

4) 2

Ans: 4

Sol.  $x-2$  is a common factor of  $x^2+ax+b$  and  $x^2+cx+d$

$$\Rightarrow 4+2a+b=0, 4+2c+d=0$$

$$\Rightarrow 2a+b=2c+d$$

$$\Rightarrow b-d=2(c-a)$$

$$\Rightarrow \frac{b-d}{c-a}=2$$

12. The solution set contained in R of the inequation  $3^x+3^{1-x}-4 < 0$  is [EAMCET 2003]

- 1) (1, 3)                      2) (0, 1)                      3) (1, 2)                      4) (0, 2)

Ans: 2

Sol.  $3^x + \frac{3}{3^x} - 4 < 0$

$$3^{2x} + 3 - 4 \cdot 3^x < 0$$

$$(3^x - 1)(3^x - 3) < 0$$

$$1 < 3^x < 3$$

$\therefore$  The solution set is (0,1)

13. The minimum value of  $2x^2+x-1$  is [EAMCET 2003]

- 1) 1/4                      2) 3/2                      3) -9/8                      4) 9/4

Ans: 3

Sol. Minimum value of  $ax^2+bx+c$  is  $\frac{4ac-b^2}{4a}$

$$\therefore \text{Minimum value of } 2x^2+x-1 \text{ is } \frac{4(2)(-1)-1}{4(2)} = -\frac{9}{8}$$

14. If the equations  $x^2+ax+b=0$  and  $x^2+bx+a=0$  ( $a \neq b$ ) have a common root then  $a+b=$

- 1) -1                      2) 2                      3) 3                      4) 4

[EAMCET 2002]

Ans: 1

Sol. Let ' $\alpha$ ' be a common root of  $x^2+ax+b=0$  and  $x^2+bx+a=0$

$$\Rightarrow \alpha^2 + a\alpha + b = 0 \quad \text{---(1)}$$

$$\alpha^2 + b\alpha + a = 0 \quad \text{---(2)} \quad \text{---(1)}$$

Solve (1) & (2)

$$(1) - (2) \Rightarrow (a-b)\alpha + (b-a) = 0$$

$$\Rightarrow \alpha = 1$$

$$1 + a + b = 0$$

$$\therefore a + b = -1$$

15. If '3' is a root of  $x^2+kx-24=0$  it is also root of [EAMCET 2002]

- 1)  $x^2+5x+k=0$     2)  $x^2+kx+24=0$     3)  $x^2-kx+6=0$     4)  $x^2-5x+k=0$

Ans: 2

Sol. Put  $x = 3$

$$9 + 3k - 24 = 0 \Rightarrow k = 5$$

Put  $k=5$  in options.

$\therefore 3$  is also a root of  $x^2 - kx + 6 = 0$

$$x^2 - 5x + 6 = 0$$

16. If  $\alpha, \beta$  are the roots of  $x^2 + bx + c = 0$  and  $\alpha + h, \beta + h$  are the roots of  $x^2 + qx + r = 0$  then  $h =$  [EAMCET 2001]

- 1)  $b+q$                       2)  $b-q$                       3)  $\frac{1}{2} (b+q)$                       4)  $\frac{1}{2} (b-q)$

Ans: 4

Sol.  $\alpha, \beta$  are the roots of  $x^2 + bx + c = 0$

$$\Rightarrow \alpha + \beta = -b$$

$\alpha + h, \beta + h$  are the roots of  $x^2 + qx + r = 0$

$$\Rightarrow (\alpha + h) + (\beta + h) = -q$$

$$\alpha + \beta + 2h = -q$$

$$-b + 2h = -q$$

$$\Rightarrow h = \frac{1}{2}(b - q)$$

17. If  $20^{3-2x^2} = (40\sqrt{5})^{3x^2-2}$  then  $x =$  [EAMCET 2001]

- 1)  $\pm \sqrt{\frac{13}{2}}$                       2)  $\pm \sqrt{\frac{12}{13}}$                       3)  $\pm \sqrt{\frac{4}{5}}$                       4)  $\pm \sqrt{\frac{5}{4}}$

Ans: 2

Sol.  $(20)^{3-2x^2} = [20 \times 2\sqrt{5}]^{3x^2-2}$

$$= [20\sqrt{20}]^{3x^2-2}$$

$$= [(20)^{3/2}]^{3x^2-2}$$

$$(20)^{3-2x^2} = (20)^{\frac{9x^2-6}{2}}$$

$$3 - 2x^2 = \frac{9x^2 - 6}{2}$$

$$6 - 4x^2 = 9x^2 - 6$$

$$13x^2 = 12$$

$$x = \pm \sqrt{\frac{12}{13}}$$

18. If  $\alpha, \beta$  are the roots of  $9x^2 + 6x + 1 = 0$  then the equation with the roots  $1/\alpha, 1/\beta$  is [EAMCET 2000]

- 1)  $2x^2 + 3x + 18 = 0$     2)  $x^2 + 6x - 9 = 0$                       3)  $x^2 + 6x + 9 = 0$                       4)  $x^2 - 6x + 9 = 0$

Ans: 3

Sol  $f\left(\frac{1}{x}\right) = 0$

$$\Rightarrow 9\left(\frac{1}{x}\right)^2 + 6\left(\frac{1}{x}\right) + 1 = 0$$

$$x^2 + 6x + 9 = 0$$

19. The equation formed by decreasing each root of  $ax^2 + bx + c = 0$  by 1  $2x^2 + 8x + 2 = 0$  is then [EAMCET 2000]

- 1)  $a = -b$                       2)  $b = -c$                       3)  $c = -a$                       4)  $b = a + c$

Ans: 2

Sol. The equation formed by decreasing each root of  $ax^2 + bx + c = 0$  by 1 is  $2x^2 + 8x + 2 = 0$

$\Rightarrow$  The equation formed by increasing each root of  $2x^2 + 8x + 2 = 0$  is  $ax^2 + bx + c = 0$

$$\therefore ax^2 + bx + c = 2(x-1)^2 + 8(x-1) + 2$$

$$ax^2 + bx + c = 2x^2 + 4x - 4$$

$$\therefore \frac{a}{2} = \frac{b}{4} = \frac{c}{-4} \Rightarrow 2a = b = -c$$

20. If  $(3 + i)$  is a root of the equation  $x^2 + ax + b = 0$  then  $a =$  [EAMCET 2000]

- 1) 3                                      2) -3                                      3) 6                                      4) -6

Ans: 4

Sol. If  $3 + i$  is one root of the given equation then the other root be  $3 - i$

Sum of the roots of  $x^2 + ax + b = 0$  is  $-a$

$$\Rightarrow 3 + i + 3 - i = -a$$

$$\therefore a = -6$$