

PROPERTIES OF TRIANGLES

PREVIOUS EAMCET BITS

1. In any ΔABC , $a(b \cos C - c \cos B) =$ [EAMCET 2009]

- 1) $b^2 + c^2$ 2) $b^2 - c^2$ 3) $\frac{1}{b} + \frac{1}{c}$ 4) $\frac{1}{b^2} - \frac{1}{c^2}$

Ans: 2

Sol. $\frac{1}{2}(2ab \cos C - 2ca \cos B)$
 $\frac{1}{2}[a^2 + b^2 - c^2 - c^2 - a^2 + b^2]$
 $= b^2 - c^2$

2. In a ΔABC $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ [EAMCET 2009]

- 1) $\cos^2 A$ 2) $\cos^2 B$ 3) $\sin^2 A$ 4) $\sin^2 B$

Ans: 3

Sol. $\frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$
 $= \frac{4\Delta^2}{b^2c^2} = \left(\frac{2\Delta}{bc}\right)^2 = \sin^2 A$

3. In ΔABC if $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$ then $C =$ [EAMCET 2008]

- 1) 90° 2) 60° 3) 45° 4) 30°

Ans: 2

Sol. Given $\frac{1}{b+c} + \frac{1}{c+a} = \frac{3}{a+b+c}$
 $\Rightarrow 1 + \frac{b}{a+c} + 1 + \frac{a}{a+c} = 3$
 $\Rightarrow b(b+c) + a(a+c) = (a+c)(b+c)$
 $\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$
 $\Rightarrow a^2 + b^2 - c^2 = ab$
 Now, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{ab}{2ab} = \frac{1}{2}$
 $\Rightarrow \underline{C} = 60^\circ$

4. Observe the following statements : [EAMCET 2008]

I) In ΔABC $b \cos^2 \frac{C}{2} + c \cos^2 \frac{B}{2} = s$

II) In ΔABC , $\cot \frac{A}{2} = \frac{b+c}{2} \Rightarrow B = 90^\circ$

Which of the following is correct?

- 1) Both I and II are true
 2) I is true, II is false
 3) I is false, II is true
 4) Both I and II are false

Ans: 2

Sol. I) $b \cos^2 \frac{C}{2} + c \cos^2 \frac{B}{2} = b \frac{s(s-c)}{ab} + c \frac{s(s-b)}{ac} = \frac{s}{a}(s-c+s-b) = s$

II) If $A = 45^\circ, B = 90^\circ, C = 45^\circ$, then $a = \sqrt{2}R, b = 2R, c = \sqrt{2}R$

But $\frac{b+c}{2} = \frac{2R + \sqrt{2}R}{2} = (\sqrt{2} + 1) \frac{R}{\sqrt{2}} = \frac{R}{\sqrt{2}} \cot 22 \frac{1}{2}^\circ \neq \cot \frac{A}{2}$

5. In a triangle, if $r_1 = 2r_2 = 3r_3$, then $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} =$ [EAMCET 2008]

- 1) $\frac{75}{60}$ 2) $\frac{155}{60}$ 3) $\frac{176}{60}$ 4) $\frac{191}{60}$

Ans: 4

Sol. $r_1 = 2r_2 = 3r_3 \Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = \frac{\Delta}{k}$ (say)

$\Rightarrow s-a = k, s-b = 2k, s-c = 3k \Rightarrow s-a+s-b+s-c = 6k$

$\Rightarrow s = 6k \Rightarrow a = 5k, b = 4k, c = 3k$

$\therefore \frac{a}{b} + \frac{b}{c} + \frac{c}{a} = \frac{5k}{4k} + \frac{3k}{5k} = \frac{5}{4} + \frac{3}{5} = \frac{75+80-36}{60} = \frac{191}{60}$

(or)

If $xr_1 = yr_2 = zr_3$, then

$a : b : c = y + z : z + x : x + y$

$\therefore a : b : c = 5 : 4 : 3$

6. If two angles of ΔABC are 45° and 60° , then the ratio of the smallest and the greatest sides are [EAMCET 2007]

- 1) $(\sqrt{3}-1):1$ 2) $\sqrt{3}:\sqrt{2}$ 3) $1:\sqrt{3}$ 4) $\sqrt{3}:1$

Ans: 1

Sol. Angles are $45^\circ, 60^\circ$ and 75° .

The ratio of smallest and greatest sides = $\sin 45^\circ; \sin 75^\circ = \sqrt{3}-1:1$

7. In ΔABC , $(a+b+c) \left(\tan \frac{A}{2} + \tan \frac{B}{2} \right) =$ [EAMCET 2007]

- 1) $2c \cot \frac{C}{2}$ 2) $2a \cot \frac{A}{2}$ 3) $2b \cot \frac{B}{2}$ 4) $\tan \frac{C}{2}$

Ans: 1

Sol. $(a+b+c) \left(\tan \frac{A}{2} + \tan \frac{B}{2} \right) = 2s \left(\frac{\Delta}{s(s-a)} + \frac{\Delta}{s(s-b)} \right)$

$= \frac{2\Delta c}{(s-a)(s-b)} = \frac{2c \sqrt{s(s-a)(s-b)(s-c)}}{(s-a)(s-b)}$

$= 2c \cot \frac{C}{2}$

8. In ΔABC , with usual notation, observe the two statements given below [EAMCET 2007]

I) $r_1 r_2 r_3 = \Delta^2$

II) $r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2$

Which of the following is correct

1) Both I and II are true

2) I is true, II is false

3) I is false, II is true

4) Both I and II are false

Ans: 1

Sol. i) $r_1 r_2 r_3 = \frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c} = \Delta^2$

ii) $r_1 r_2 + r_2 r_3 + r_3 r_1 = \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} + \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c} + \frac{\Delta}{s-c} \cdot \frac{\Delta}{s-a}$
 $= s(s-c) + s(s-a) + s(s-b) = s^2$

9. If, in a ΔABC , $\tan \frac{A}{2} = \frac{5}{6}$ and $\tan \frac{C}{2} = \frac{2}{5}$ then a, b, c are such that : [EAMCET 2006]

1) $b^2 + ac$

2) $2b = a + c$

3) $2ac = b(a + c)$

4) $a + b = c$

Ans: 2

Sol. $\tan \frac{A}{2} \cdot \tan \frac{C}{2} = \frac{5}{6} \cdot \frac{2}{5} = \frac{1}{3}$

$$\sqrt{\frac{(s-b)(s-c)(s-a)(s-b)}{s(s-a)s(s-c)}} = \frac{1}{3}$$

$$\Rightarrow \frac{s-b}{s} = \frac{1}{3} \Rightarrow 3s - 3b = s$$

$$2s = 3b$$

$$a + b + c = 3b$$

$$\Rightarrow a + c = 2b$$

10. The angles of a triangle are in the ratio 3 : 5 : 10. Then the ratio of the smallest side to the greatest side is [EAMCET 2006]

1) $1 : \sin 10^\circ$

2) $1 : 2 \sin 10^\circ$

3) $1 : \cos 10^\circ$

4) $1 : 2 \cos 10^\circ$

Ans: 4

Sol. Let angles as $3x, 5x, 10x$

$$\therefore 18x = 180^\circ \Rightarrow x = 10^\circ$$

$$\therefore \text{angle are } 30^\circ, 50^\circ, 100^\circ$$

$$a : c = \sin A : \sin C = \sin 30 : \sin 100$$

$$\frac{1}{2} : \sin(90+10) = 1 : 2 \cos 10^\circ$$

11. In a triangle ABC, $\frac{s-a}{\Delta} = \frac{1}{8}, \frac{s-b}{\Delta} = \frac{1}{12}, \frac{s-c}{\Delta} = \frac{1}{24}$ then b = [EAMCET 2006]

1) 16

2) 20

3) 24

4) 28

Ans: 1

Sol. $\frac{1}{r_1} = \frac{s-a}{\Delta} = \frac{1}{8}; \quad \frac{1}{r_2} = \frac{s-b}{\Delta} = \frac{1}{12}; \quad \frac{1}{r_3} = \frac{s-c}{\Delta} = \frac{1}{24}$

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{6}{24} = \frac{1}{4}$$

$$(r_2 - r)(r_1 + r_3) = b^2$$

$$(12 - 4)(24 + 8) = b^2 \Rightarrow 16 \times 16, b = 16$$

12. In a ΔABC , $a(\cos^2 B + \cos^2 C) + \cos A(c \cos C + b \cos B) =$ [EAMCET 2005]

- 1) a 2) b 3) c 4) a + b + c

Ans: 1

Sol.
$$= a \left[\left(\frac{a^2 + c^2 - b^2}{2ac} \right)^2 + \left(\frac{a^2 + b^2 - c^2}{2ab} \right)^2 \right]$$

$$+ \left(\frac{b^2 + c^2 - a^2}{2bc} \right) \left[c \left(\frac{b^2 + a^2 - c^2}{2ab} \right) + b \left(\frac{a^2 + c^2 - b^2}{2ac} \right) \right]$$

$$\frac{a^2 + c^2 - b^2}{2a} + \frac{a^2 + b^2 - c^2}{2a} = \frac{2a^2}{2a} = a$$

13. In a ΔABC , $\sum (b+c) \tan \frac{A}{2} \tan \left(\frac{B-C}{2} \right) =$ [EAMCET 2005]

- 1) a 2) b 3) c 4) 0

Ans: 4

Sol. From Napier's formula $\tan \left(\frac{B-C}{2} \right) = \frac{b-c}{b+c} \cot \frac{A}{2}$

$$\sum (b+c) \tan \frac{A}{2} \tan \left(\frac{B-C}{2} \right) = \sum (b+c) \tan \frac{A}{2} \cdot \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$\sum (b-c) = 0$$

14. Two sides of a triangle are given by the roots of the equation $x^2 - 5x + 6 = 0$ and the angle between the sides is $\pi/3$. Then the perimeter of the triangle is [EAMCET 2005]

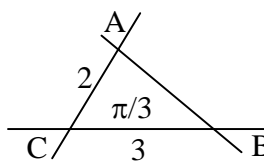
- 1) $5 + \sqrt{2}$ 2) $5 + \sqrt{3}$ 3) $5 + \sqrt{5}$ 4) $5 + \sqrt{7}$

Ans: 4

Sol. $c^2 = a^2 + b^2 - 2ab \cos C$

$$= 9 + 4 - 12 \times \frac{1}{2} = 7$$

$$= c = \sqrt{7}$$



$$2S = a + b + c = 5 + \sqrt{7}$$

15. If, in a ΔABC , $r_3 = r_1 + r_2 + r$, then $\angle A + \angle B =$ [EAMCET 2004]

- 1) 120° 2) 100° 3) 90° 4) 80°

Ans: 3

Sol. $r_3 - r = 4R \sin^2 \frac{C}{2}$

$$r_1 + r_2 = 4R \cos^2 \frac{C}{2}$$

$$r_1 + r_2 + r - r_3 = 0$$

$$\Rightarrow 4R \left(\cos^2 \frac{c}{2} - \sin^2 \frac{c}{2} \right) = 0$$

$$\Rightarrow 4R \cos c = 0 \Rightarrow c = 90^\circ$$

$$\therefore \underline{A} + \underline{B} = 90^\circ$$

16. In a ΔABC , $(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2} =$ [EAMCET 2004]

1) a^2 2) c^2 3) b^2 4) $a^2 + b^2$

Ans: 2

Sol. $a^2 + b^2 - 2ab \left(\cos^2 \frac{c}{2} - \sin^2 \frac{c}{2} \right) = c^2$

17. In a ΔABC , the correct formulae among the following are [EAMCET 2004]

I) $r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

II) $r_1 = (s-a) \tan \frac{A}{2}$

III) $r_3 = \frac{\Delta}{s-c}$

1) only I, II 2) only II, III 3) only I, III 4) I, II, III

Ans: 3

Sol. I is true, II is false, III is true

18. If in a ΔABC , $r_1 < r_2 < r_3$ then [EAMCET 2003]

1) $a < b < c$ 2) $a > b > c$ 3) $b < a < c$ 4) $a < c < b$

Ans: 1

Sol. $r_1 < r_2 < r_3$

$$\Rightarrow \frac{\Delta}{s-a} < \frac{\Delta}{s-b} < \frac{\Delta}{s-c}$$

$$\Rightarrow s-a > s-b > s-c$$

$$\Rightarrow -a > -b > -c$$

$$\therefore a < b < c$$

19. If in a ΔABC , if $3a = b+c$, then $\cot \frac{B}{2} \cdot \cot \frac{C}{2} =$ [EAMCET 2003]

1) 1 2) 2 3) 3 4) 4

Ans: 2

Sol. $\cot \frac{B}{2} \cot \frac{C}{2} = \frac{s(s-b)}{\Delta} \times \frac{s(s-c)}{\Delta}$
 $= \frac{s}{s-a} = \frac{2s}{2(s-a)} = \frac{a+b+c}{b+c-a} = 2$

20. In a ΔABC , if $b = 20$, $c = 21$ and $\sin A = 3/5$, then $a = \dots\dots\dots$ [EAMCET 2003]

1) 12 2) 13 3) 14 4) 15

Ans: 2

Sol. $\sin A = \frac{3}{5} \Rightarrow \cos A = \frac{4}{5}$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a = 13$$

21. If ΔABC is right angled at A, then $r_2 + r_3 =$ [EAMCET 2002]

- 1) $r_1 - r$ 2) $r_1 + r$ 3) $r - r_1$ 4) R

Ans: 1

Sol. $\angle A = 90^\circ$ and $r_2 + r_3$

$$= 4R \cos^2 \frac{A}{2} = 2R$$

$$r_2 - r = 4R \sin^2 \frac{A}{2} = 2R$$

22. In a ΔABC , $\frac{\cos C + \cos A}{c+a} + \frac{\cos B}{b} =$ [EAMCET 2001]

- 1) $\frac{1}{a}$ 2) $\frac{1}{b}$ 3) $\frac{1}{c}$ 4) $\frac{c+a}{b}$

Ans: 2

Sol. $\frac{\cos C + \cos A}{c+a} + \frac{\cos B}{b} =$

$$= \frac{b \cos C + b \cos A + c \cos B + a \cos B}{b(c+a)}$$

$$\frac{(c+a)}{b(c+a)} = \frac{1}{b}$$

23. In a ΔABC , $a^2 \sin 2C + c^2 \sin 2A$ [EAMCET 2001]

- 1) Δ 2) 2Δ 3) 3Δ 4) 4Δ

Ans: 4

Sol. $a^2 \sin 2C + c^2 \sin 2A$

$$= 4R^2 \sin^2 A \cdot 2 \sin C \cos C + 4R^2 \sin^2 C \cdot 2 \sin A \cos A$$

$$= 8R^2 \sin A \sin C (\sin A \cos C + \cos A \sin C)$$

$$= 8R^2 \sin A \sin B \sin C = 4\Delta$$

24. If in a ΔABC , a, b, c are in arithmetic progression, then the $(A/2) \tan (C/2) =$ [EAMCET 2000]

- 1) $1/4$ 2) $1/3$ 3) 3 4) 4

Ans: 2

Sol. $\tan \frac{A}{2} \tan \frac{C}{2} = \frac{\Delta}{s(s-a)} \cdot \frac{\Delta}{s(s-c)}$

$$= \frac{s-b}{s} = \frac{2(s-b)}{2s}$$

$$= \frac{a+c-b}{a+b+c} = \frac{2b-b}{2b+b} = \frac{1}{3}$$

25. If a ΔABC , $\cos A + \cos B + \cos C =$ [EAMCET 2000]

- 1) $1 + \frac{r}{R}$ 2) $1 - \frac{r}{R}$ 3) $1 - \frac{R}{r}$ 4) $1 + \frac{R}{r}$

Ans: 1

Sol. $\cos A + \cos B + \cos C = 1 + 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$
 $= 1 + \frac{4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}}{R} = 1 + \frac{r}{R}$

26. In a ΔABC , $r + r_3 + r_1 - r_2 =$ **[EAMCET 2000]**
 1) $4R \cos A$ 2) $4R \cos B$ 3) $4R \cos C$ 4) $4R$
 Ans: 2

Sol. $r_1 + r_3 = 4R \cos^2 \frac{B}{2}$
 $r_2 - r_1 = 4R \sin^2 \frac{B}{2}$
 $r + r_1 + r_3 - r_2 = 4R \left(\cos^2 \frac{B}{2} - \sin^2 \frac{B}{2} \right) = 4R \cos B$



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