PARTIAL DIFFERENTIATION

PREVIOUS EAMCET BITS

1. $z = \tan(y+ax) + \sqrt{y-ax} \Rightarrow z_{xx} - a^2 z_{yy} =$ **[EAMCET 2009]**

1) 0 2) 2 3) $z_x + z_y$ 4) $z_x z_y$

Ans: 1

Sol. $Z = f(y+ax) + g(y-ax)$

$\Rightarrow Z_{xx} - a^2 Z_{yy} = 0$

2. If $z = \sec^{-1}\left(\frac{x^4+y^4-8x^2y^2}{x^2+y^2}\right)$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} =$ **[EAMCET 2008]**

1) cot z 2) 2 cotz 3) 2tanz 4) 2 secz

Ans: 2

Sol. z = $z = \sec^{-1}\left(\frac{x^4+y^4-8x^2y^2}{x^2+y^2}\right) \Rightarrow \sec z$ is a homogeneous function of degree 2

$\Rightarrow x\frac{\partial}{\partial x}[\sec z] + y\frac{\partial}{\partial y}[\sec z] = 2\sec z \Rightarrow x\sec z\tan z\frac{\partial z}{\partial x} + y\sec z\tan z\frac{\partial z}{\partial y} = 2\sec z$

$\Rightarrow x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z$

3. If $z = \log(\tan x + \tan y)$, then $(\sin 2x)\frac{\partial z}{\partial x} + (\sin 2y)\frac{\partial z}{\partial y} =$ **[EAMCET 2007]**

1) 1 2) – 2 3) 3 4) 4

Ans: 2

Sol. $\frac{\partial z}{\partial x} = \frac{\sec^2 x}{\tan x + \tan y}$

$\frac{\partial z}{\partial y} = \frac{\sec^2 y}{\tan x + \tan y}$

$\sin 2x\frac{\partial z}{\partial x} + \sin 2y\frac{\partial z}{\partial y} = 2$

4. $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right) \Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ **[EAMCET 2006]**

1) sin u 2) tan u 3) cos u 4) cot u

Ans: 2

Sol. $\sin u = \frac{x^2+y^2}{x+y} = f(x,y)$

f(x, y) is a Homogeneous equation of '1' degree

$\therefore$ by Euler's theorem

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=n.f.=f(x,y)$$

$$x\frac{\partial u}{\partial x}+y.\frac{\partial u}{\partial y}=n.\frac{f(u)}{f'(u)}$$

$$\Rightarrow \frac{\sin u}{\cos u}=\tan u$$

5 $z=\cos^{-1}(x+y)+\sec^{-1}(y+2x)\Rightarrow \frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial y^2}=$ **[EAMCET 2005]**

1) $\frac{3\partial^2 z}{\partial x\partial y}$ 2) $\frac{-\partial^2 z}{\partial x\partial y}$ 3) $\frac{-3\partial^2 z}{\partial x\partial y}$ 4) $\frac{\partial^2 z}{\partial x\partial y}$

Ans: 1

Sol. differentiating partially

6. $f(x,y)=2(x-y)^2-x^4-y^4\Rightarrow \left(f_{xx}f_{yy}-f_{xy}^2\right)\Big|_{(0,0)}$ **[EAMCET 2004]**

1) 32 2) 16 3) 0 4) – 1

Ans: 3

Sol. $f(x,y)=2(x-y)^2-x^4-y^4$

$(f_{xx})_{(0,0)}=4-12x^2=4$

$(f_{yy})_{(0,0)}=-4-12y^2$

$(f_{yy})_{(0,0)}=-4$

$\therefore \left(f_{xx}.f_{yy}-f_{yy}^2\right)_{(0,0)}=4\times 4-4^2=0$

7. If $u(x,y)=y\log x+x\log y$, then $u_x u_y-u_x\log x-u_y\log y+\log x\log y=...$ **[EAMCET 2003]**

1) 0 2) – 1 3) 1 4) 2

Ans: 3

Sol. $u(x,y)=y\log x+x\log y$

$u_x=\frac{y}{x}+\log y$ and $u_y=\log x+\frac{x}{y}$

$u_x u_y-u_x\log x-u_y\log y+\log x\log y$

$=(u_x-\log y)(u_y-\log x)$

$=\frac{y}{x}.\frac{x}{y}=1$

8. If $z=\frac{y}{x}\left[\sin\frac{x}{y}+\cos\left(1+\frac{y}{x}\right)\right]$, then $x\frac{\partial z}{\partial x}=$ **[EAMCET 2002]**

1) $y\frac{\partial z}{\partial y}$ 2) $-y\frac{\partial z}{\partial y}$ 3) $2y\frac{\partial z}{\partial y}$ 4) $2y\frac{\partial z}{\partial x}$

Ans: 2

Sol. $z=\frac{y}{x}\left[\sin\frac{x}{y}+\cos\left(1+\frac{y}{x}\right)\right]$ then

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0\Rightarrow x\frac{\partial z}{dx}=-y\frac{\partial z}{dy}$

9. If $z=\sec(y-ax)+\tan(y+ax)$, then $\frac{\partial^2 z}{\partial x^2}-a^2\frac{\partial^2 z}{\partial y^2}=$ **[EAMCET 2002]**

1) z 2) 2z 3) 0 4) – z

Ans: 3

Sol. $z=\sec(y-ax)+\tan(y+ax)$

$z=f(ax+y)+\phi(y-ax)$

$\frac{\partial^2 z}{\partial x^2}+a^2\frac{\partial^2 z}{\partial y^2}$

$\therefore\ \frac{\partial^2 z}{\partial x^2}+a^2\frac{\partial^2 z}{\partial y^2}\Rightarrow\frac{\partial^2 z}{\partial x^2}-a^2\frac{\partial^2 z}{\partial y^2}=0$

10. If $u=e^{-x^2-y^2}$, then **[EAMCET 2001]**

1) $xu_x=yu_y$ 2) $yu_x=xu_y$ 3) $yu_x+xu_y=0$ 4) $x^2u_y+y^2u_x=0$

Ans: 2

Sol. $u=e^{-x^2-y^2}$

$ux=-2xe^{-x^2-y^2};uy=-2ye^{-x^2-y^2}$

$\therefore\ yu_x=xu_y$

11. If $u=xy^2\tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ **[EAMCET 2001]**

1) 2u 2) u 3) 3u 4) $\frac{1}{3}u$

Ans: 3

Sol. Given $u=xy^2.\tan^{-1}\left(\frac{y}{x}\right)$

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu=3u(\because n=3)$

12. If $u=\log_e(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$, then $\frac{\partial^2 y}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=$ **[EAMCET 2000]**

1) 0 2) 2u 3) 1/u 4) u

Ans: 1

Sol. $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$

13. If $u=\cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=$ **[EAMCET 2000]**

1) $\frac{1}{2}\cot u$ 2) $2\cot u$ 3) $-\frac{1}{2}\cot u$ 4) $3\cot u$

Ans: 3

Sol. $n=\frac{1}{2}\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-n\cot u$

$=-\frac{1}{2}\cot u$

★ ★ ★ ★