

# MULTIPLE AND SUBMULTIPLE ANGLES

## PREVIOUS EAMET BITS

1. If  $A = 35^\circ$ ,  $B = 15^\circ$  and  $C = 40^\circ$ , then  $\tan A \tan B + \tan B \tan C + \tan C \tan A$  [EAMCET 2008]  
 1) 0                                      2) 1                                      3) 2                                      4) 3

Ans: 2

Sol.  $A + B + C = 35^\circ + 15^\circ + 40^\circ = 90^\circ$   
 $\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

2. If  $\tan \theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$ , then which of the following is equal to 1? [EAMCET 2008]

- 1)  $\tan 2\theta$                                       2)  $\tan 3\theta$                                       3)  $\tan^2 \theta$                                       4)  $\tan^3 \theta$

Ans: 2

Sol.  $\tan \theta + \tan\left(\theta + \frac{\pi}{3}\right) + \tan\left(\theta + \frac{2\pi}{3}\right) = 3$

$$\Rightarrow \tan \theta + \frac{\tan \theta + \tan\left(\frac{\pi}{3}\right)}{1 - \tan \theta \tan\left(\frac{\pi}{3}\right)} + \frac{\tan \theta + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan \theta \tan\left(\frac{2\pi}{3}\right)} = 3$$

$$\Rightarrow \tan \theta + \frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} + \frac{\tan \theta - \sqrt{3}}{1 - \sqrt{3} \tan \theta} = 3$$

$$\Rightarrow \tan \theta + \frac{\tan \theta + \sqrt{3} \tan^2 \theta + \sqrt{3} + 3 \tan \theta + \tan \theta - \sqrt{3} \tan^2 \theta + 3 \tan \theta}{(1 - \sqrt{3} \tan \theta)(1 + \sqrt{3} \tan \theta)}$$

$$\Rightarrow \tan \theta + \frac{8 \tan \theta}{1 - 3 \tan^2 \theta} = 3 \Rightarrow \frac{\tan \theta - 3 \tan^3 \theta + 8 \sin \theta}{1 - 3 \tan^2 \theta} = 3$$

$$\Rightarrow 3 \left[ \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right] = 3 \Rightarrow \tan 3\theta = 1$$

3. If  $x = \tan 15^\circ$ ,  $y = \operatorname{cosec} 75^\circ$  and  $z = 4 \sin 18^\circ$ , then [EAMCET 2006]  
 1)  $x > y > z$                                       2)  $y < z < x$                                       3)  $z < x < y$                                       4)  $x < z < y$

Ans: 1

Sol.  $x = \tan 15^\circ = 2 - \sqrt{3} = 0.3$

$$y = \operatorname{cosec} 75^\circ = \frac{1}{\sin 75^\circ} = \frac{2\sqrt{2}}{\sqrt{3} + 1}$$

$$= \sqrt{6} - \sqrt{2} \cong 0.9$$

$$z = 4 \sin 18^\circ = \sqrt{5} - 1 = 1.05$$

$$x < y < z$$

4.  $\frac{\tan 3A}{\tan A} = \alpha \Rightarrow \frac{\sin 3A}{\sin A} =$  [EAMCET 2005]

- 1)  $\frac{2a}{a+1}$                                       2)  $\frac{2a}{a-1}$                                       3)  $\frac{a}{a+1}$                                       4)  $\frac{a}{a-1}$

Ans: 2

Sol.  $a = \frac{\tan 3A}{\tan A}$ , put  $A = 45^\circ \Rightarrow a = -1$

Verification  $\frac{\sin 3A}{\sin A} = 1 = \frac{2a}{a-1}$

5.  $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ =$   
 1) 4                                      2) 3                                      3) 2                                      4) 1

[EAMCET 2004]

Ans: 1

Sol.  $\tan 9^\circ - \tan 27^\circ - \cot 27^\circ + \cos 9^\circ = \left( \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ} \right) - \left( \frac{\sin 27^\circ}{\cos 27^\circ} + \frac{\cos 27^\circ}{\sin 27^\circ} \right)$   
 $\Rightarrow \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = \frac{2 \times 4}{\sqrt{5}-1} - \frac{2 \times 4}{\sqrt{5}+1} = 4$

6.  $\cos 6^\circ \sin 24^\circ \cos 72^\circ =$

1)  $-1/8$                                       2)  $-1/4$                                       3)  $1/8$                                       4)  $1/4$

[EAMCET 2000]

Ans: 3

Sol.  $\frac{1}{2}(2 \cos 6^\circ \sin 24^\circ) \cos 72^\circ$   
 $= \frac{1}{2}(\sin 30^\circ + \sin 18^\circ) \cos 72^\circ$   
 $= \frac{1}{2} \left( \frac{1}{2} + \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{5}-1}{4} \right) = \frac{1}{8}$

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