MAXIMA AND MINIMA
PREVIOUS EAMCET BITS

1. The function $f(x) = x^3 + ax^2 + bx + c, a^2 \le 3b$ has **[EAMCET 2009]**

   1) one maximum value 2) one minimum value
   3) no extreme value 4) one maximum and one minimum value

   Ans: 3

Sol. $f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

$\therefore\ \Delta = 4a^2 - 12b = 4(a^2 - 3b) < 0$

$\therefore\ f'(x)$ never zero.

2. The maximum value of $\frac{\log x}{x}, 0 < x < \infty$ is **[EAMCET 2009]**

   1) ∞ 2) e 3) 1 4) e^{-1}

   Ans: 4

Sol. $f(x) = \frac{\log x}{x}$

$\therefore\ f'(x) = \frac{x\left(\frac{1}{x}\right) - \log x(1)}{x^2} = 0 \Rightarrow f'(x) = \frac{1 - \log x}{x} = 0$

$\Rightarrow x = e$

$\therefore$ Max. $f = \frac{1}{e}$

3. If m and M respectively denote the minimum and maximum of $f(x) = (x-1)^2 + 3$ for $x \in [-3, 1]$ then the order pair (m, M) **[EAMCET 2008]**

   1) (-3, 19) 2) (3, 19) 3) (–19, 3) 4) (–19, –3)

   Ans: 2

Sol. $f(x) = (x-1)^2 + 3 \Rightarrow f'(x) = 2(x-1) \Rightarrow f''(x) = 2 > 0$

$f'(x) = 0 \Rightarrow x = 1$. Minimum value = f(1) = 3

$-3 \le x \le 1 \Rightarrow -4 \le x - 1 \le 0 \Rightarrow (x-1)^2 + 3 \le 19$

Minimum value = 3 $\therefore$ (m, M) = (3, 19)

4. The condition for $f(x) = x^3 + px^2 + qx + r\ (x \in \mathbb{R})$ to have no extreme value, is **[EAMCET 2007]**

   1) $p^2 < 3q$ 2) $2p^2 < q$ 3) $p^2 < 1/4q$ 4) $p^2 > 3q$

   Ans: 1

Sol. f(x) has no extreme value then $f'(x) = 0$ have no real roots.

$f'(x) = 3x^2 + 2px + q$

$4p^2 - 12q < 0 \Rightarrow p^2 < 3q$

5. Observe the statements given below: **[EAMCET 2007]**

Assertion (A) : $f'(x) = xe^{-x}$ has the maximum at x = 1

Reason (R) : $f'(1) = 0$ and $f''(1) < 0$

Which of the following is correct ?

1) Both A and R are true and R is the correct reason for A
2) Both A and R are true , but R is not the correct reason for A
3) A is true, R is false
4) A is false, R is true

Ans: 1

Sol. $f(x) = xe^{-x} \Rightarrow f'(x) = (1-x)e^{-x}$

$f'(x) = 0 \Rightarrow x = 1$

$f''(x) = (2-x)e^{-x} \Rightarrow f''(1) = -\frac{1}{e} < 0$

$\therefore$ f(x) has maximum at x = 1

6. In the interval (- 3, 3) the function $f(x) = \frac{x}{3} + \frac{3}{x}, x \neq 0$ is

1) increasing
2) decreasing
3) neither increasing nor decreasing
4) partly increasing and partly decreasing

Ans:

Sol. $f(x) = \frac{x}{3} + \frac{3}{x},\ x \neq 0$

$f'(x) = \frac{1}{3} - \frac{3}{x^2} = 0$

$f'(x) = \frac{x^2 - 9}{3x^2}$

$x^2 - 9 < 0,\ \forall x \in (-3, 3)$

i.e., $f'(x) < 0\ \forall x \in (-3, 3)$

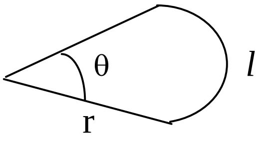
$\therefore$ decreasing

7. The perimeter of a sector is a constant. If its area is to be maximum, the sectorial angel is **[EAMCET 2006]**

1) $\frac{\pi^c}{6}$
2) $\frac{\pi^c}{4}$
3) 4^c
4) 2^c

Ans: 4

Sol. $\ell + 2r = k(\text{say})$



but $\ell = r\theta$

$r\theta + 2r = k$

$r = \dfrac{k}{\theta+2}$

Area $A = \dfrac{1}{2}r^2\theta = \dfrac{k^2\theta}{2(\theta+2)^2}$

$A^1 = \dfrac{k^2}{2}\left\{\dfrac{2(\theta+2)\theta-(\theta+2)^2}{(\theta+2)^4}\right\} = 0$

$\Rightarrow \theta = 2^c$

8. Observe the following statements : **[EAMCET 2005]**

A : $f'(x) = 2x^3 - 9x^2 + 12x - 3$ is increasing in $[-\infty, 1] \cup [2, \infty]$

R : $f'(x) < 0$ for $x \in (1, 2)$

Then which of the following is true?

1) Both A and R are true and R is the correct reason for A
2) Both A and R are true but R is not the correct reason for A
3) A is true, R is false
4) A is false, R is true

Ans: 4

Sol. $A : f'(x) = 6x^2 - 18x + 12 > 0$

$\Rightarrow x \in (-\infty, 1) \cup (2, \infty)$

$\therefore$ A is false

$B : f'(x) < 0, \forall x \in (1, 2)$ True

9. The minimum value of $2x^2 + x - 1$ is **[EAMCET 2003]**

1) $\dfrac{1}{4}$ 2) $\dfrac{3}{2}$ 3) $-\dfrac{9}{8}$ 4) $\dfrac{9}{4}$

Ans: 3

Sol. Min. value = $\dfrac{4ac-b^2}{4a} = \dfrac{-9}{8}$

10. If $\log(1+x) - \dfrac{2x}{2+x}$ is increasing then ….. **[EAMCET 2002]**

1) $0 < x < \infty$ 2) $-\infty < x < 0$ 3) $-\infty < x < \infty$ 4) $1 < x < 2$

Ans: 1

Sol. $f(x) = \log(1+x) - \dfrac{2x}{2+x}$

$f'(x) = \dfrac{1}{1+x} - \dfrac{(2+x)^2 - 2x(1)}{(2+x)^2} > 0$

$\Rightarrow \frac{x^2}{(1+x)(2+x)^2} > 0$ $\therefore$ $x > 0$

$\Rightarrow 0 < x < \infty$

11. The minimum value of $(x-\alpha)(x-\beta)$ is **[EAMCET 2001]**

1) 0 2) $\alpha\beta$ 3) $\frac{1}{4}(\alpha-\beta)^2$ 4) $\frac{-1}{4}(\alpha-\beta)^2$

Ans: 4

Sol. $x^2-(\alpha+\beta)x+\alpha\beta$

The min. value of ax^2+bx+c is $\frac{4ac-b^2}{4a}$

$\therefore \frac{4\alpha\beta-(\alpha+\beta)^2}{4} = \frac{-(\alpha-\beta)^2}{4}$

12. The maximum value of xy subject to x + y = 7 is **[EAMCET 2001]**

1) 12 2) 10 3) 49/4 4) 55/4

Ans: 3

Sol. Given x + y = 7

xy = x(7 – x)

Let $f(x) = x(7-x) \Rightarrow f'(x) = 7-2x = 0$

$\Rightarrow x = \frac{7}{2}; y = \frac{7}{2}$

13. The minimum value of x^x is **[EAMCET 2000]**

1) e 2) e^e 3) $e^{-1/e}$ 4) $e^{1/e}$

Ans:

Sol. $y = x^x \Rightarrow \log y = x\log x \Rightarrow \frac{dy}{dx} = x^x\ (1+\log x)$

$(1+\log x) = 0 \Rightarrow x = e^{-1}$

$\therefore$ Min.value = $e^{-1/e}$

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