MATHEMATICAL INDUCTION
PREVIOUS EAMCET BITS

1. Using mathematical induction, the numbers a_n 's are defined by $a_0 = 1, a_{n+1} = 3n^2 + n + a_n \left(n \ge 0\right)$, then a_n = **[EAMCET 2009]**

1) $n^3 + n^2 + 1$ 2) $n^3 - n^2 + 1$ 3) $n^3 - n^2$ 4) $n^3 + n^2$

Ans: 2

Sol. $a_0 = 1, a_1 = 1, a_2 = 3 + 1 + a_1 = 5$ and so on. Verify (2) is correct

2. For any integer $n \ge 1$, then sum $\sum_{k=1}^{n} k\left(k+2\right)$ is equal to **[EAMCET 2008]**

1) $\frac{n\left(n+1\right)\left(n+2\right)}{6}$ 2) $\frac{n\left(n+1\right)\left(2n+1\right)}{6}$ 3) $\frac{n\left(n+1\right)\left(2n+7\right)}{6}$ 4) $\frac{n\left(n+1\right)\left(2n+9\right)}{6}$

Ans: 3

Sol. $\sum_{k=1}^{n} k\left(k+2\right) = \sum_{k=1}^{n}\left(k^2 + 2k\right) = \sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k = \frac{n\left(n+1\right)\left(2n+1\right)}{6} + \frac{2n\left(n+1\right)}{2}$

$\frac{n\left(n+1\right)}{6}\left[2n+1+6\right] = \frac{n\left(n+1\right)\left(2n+7\right)}{6}$

3. If $S_n = 1^3 + 2^3 + ... + n^3$ and $T_n = 1 + 2 + ... + n$ then **[EAMCET 2007]**

1) $S_n = T_n^3$ 2) $S_n = T_n^2$ 3) $S_n = T_n^5$ 4) $S_n = T_n^4$

Ans: 2

Sol. $S_n = \sum n^3 = \frac{n^2\left(n+1\right)^2}{4}\left(\sum n\right)^2 = T_n^2$

4. For all integers $n \ge 1$, which of the following is divisible by 9 **[EAMCET 2006]**

1) $8^n + 1$ 2) $4^n - 3n - 1$ 3) $3^{2n} + 3n + 1$ 4) $10^n + 1$

Ans: 2

Sol. by verification n = 2

5. $\left\{n\left(n+1\right)\left(2n+1\right) : n \in Z\right\} \subset$ **[EAMCET 2005]**

1) $\left\{6k : k \in Z\right\}$ 2) $\left\{12k : k \in Z\right\}$ 3) $\left\{18k : k \in Z\right\}$ 4) $\left\{24k : k \in Z\right\}$

Ans: 1

Sol. $n\left(n+1\right)\left(2n+1\right)$

$= 6.\frac{n\left(n+1\right)\left(2n+1\right)}{6} = 6k, k \in Z \ \left[\because \sum n^2 \text{ is an integer}\right]$

6. $\sum_{k=1}^{5} \frac{1^3 + 2^3 + ... + k^3}{1 + 3 + 5 + ... + \left(2k-1\right)}$ **[EAMCET 2004]**

1) 22.5 2) 24.5 3) 28.5 4) 32.5

Ans: 1

Sol. $\sum_{k=1}^{5} \frac{k^2\left(k+1\right)^2}{4k^2}$

$= \frac{2^2 + 3^2 + 4^2 + 5^2 + 6^2}{4} = 22.5$

7. If $t_1 = \frac{1}{4}(n+2)(n+3)$ for n = 1, 2, …….then $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + ... + \frac{1}{t_{2003}} =$ **[EAMCET 2003]**

1) $\frac{4006}{3006}$ 2) $\frac{4003}{3007}$ 3) $\frac{4006}{3008}$ 4) $\frac{4006}{3009}$

Ans: 4

Sol. $\frac{1}{t_n} = \frac{4}{(n+2)(n+3)}$

$= 4\left[\frac{1}{n+2} - \frac{1}{n+3}\right]$

$\therefore \frac{1}{t_1} + \frac{1}{t_2} + ... + \frac{1}{t_n}$

$4\left[\frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + ... + \frac{1}{n+2} - \frac{1}{n+3}\right]$

$= \frac{4n}{3(n+3)} = \frac{4(2003)}{3(2006)} = \frac{4006}{3009}$

8. In the sequence {1}, {2, 3}, {4, 5, 6}, {7, 8, 9, 10}…….. of sets the sum of elements in the 50th set is **[EAMCET 2002]**

1) 62525 2) 65225 3) 56255 4) 55625

Ans: 1

Sol. The sum of elements in the nth set is

$S_n = \frac{n(n^2+1)}{2}$

$\therefore S_{50} = \frac{50(50^2+1)}{2} = 62525$

9. If $a_k = \frac{1}{k(k+1)}$, for $k = 1, 2, 3, ...n$ then $\left(\sum_{k=1}^{n} a_k\right)^2 =$ **[EAMCET 2000]**

1) $\frac{n}{n+1}$ 2) $\frac{n^2}{(n+1)^2}$ 3) $\frac{n^4}{(n+1)^4}$ 4) $\frac{n^6}{(n+1)^6}$

Ans: 1

Sol. Given $a_k = \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$

$= 1 + \frac{1}{2} + \frac{1}{3} + ... \left[\frac{1}{2} + \frac{1}{3} + ...\right]$

$\frac{1}{k+1} = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$

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