LIMITS

PREVIOUS EAMCET BITS

1. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+2}\right)^{x+3} =$ **[EAMCET 2009]**

1) e 2) e^2 3) e^3 4) e^5

Ans: 3

Sol. $e^{\underset{x\to\infty}{\mathrm{lt}}(x+3)\left[\frac{x+5}{x+2}-1\right]} = e^{\underset{x\to\infty}{\mathrm{Lt}}\frac{3x+9}{x+2}} = e^3\left[\because \underset{x\to\infty}{\mathrm{Lt}}\left[f(x)\right]^{g(x)} = e^{\underset{x\to\infty}{\mathrm{Lt}}\, g(x)\left[f(x)-1\right]}\right]$

2. $\lim\limits_{x\to 0}\dfrac{(1-e^x)\sin x}{x^2+x^3}$ **[EAMCET 2008]**

1) – 1 2) 0 3) 1 4) 2

Ans: 1

Sol. $\underset{x\to 0}{\mathrm{Lt}}\dfrac{(1-e^x)\sin x}{x^2+x^3} = \underset{x\to 0}{\mathrm{Lt}}\dfrac{1-e^x}{x+x^2}\times\dfrac{\sin x}{x} = \underset{x\to 0}{\mathrm{Lt}}\dfrac{1-e^x}{x+x^2} = \underset{x\to 0}{\mathrm{Lt}}\dfrac{-e^x}{1+2x} = -1$

3. If $f:R\to R$ is defined by $f(x) = [x-3]+|x-4|$ for $x\in R$ then $\lim\limits_{x\to 3-} f(x) =$ **[EAMCET 2008]**

1) – 2 2) – 4 3) – 6 4) – 8

Ans: 3

Sol. $\underset{x\to 3-}{\mathrm{Lt}}\, f(x) = \underset{x\to 3-}{\mathrm{Lt}} - \{[x-3]+|x-4|\} = -1+1 = 0$

4. If f(2) = 4 and $f'(2) = 1$ then $\lim\limits_{x\to 2}\dfrac{xf(2)-2f(x)}{x-2} =$ **[EAMCET 2008]**

1) – 2 2) 1 3) 2 4) 3

Ans: 3

Sol. $\underset{x\to 2}{\mathrm{Lt}}\dfrac{xf(2)-2f(x)}{x-2} = \underset{x\to 2}{\mathrm{Lt}}\dfrac{f(2)-2f'(2)}{1} = \dfrac{4-2}{1} = 2$

5. $\lim\limits_{x\to 0}\dfrac{e^x-e^{\sin x}}{2(x-\sin x)} =$ **[EAMCET 2007]**

1) $-\dfrac{1}{2}$ 2) $\dfrac{1}{2}$ 3) 1 4) $\dfrac{3}{2}$

Ans: 2

Sol. $\lim\limits_{x\to 0}\dfrac{e^{\sin x}\left(e^{x-\sin x}-1\right)}{2(x-\sin x)} = \dfrac{1}{2}$

6. If $f(x) = \begin{cases}\dfrac{\sin(1+[x])}{[x]} & \text{for } [x]\neq 0\\ 0 & \text{for } [x]=0\end{cases}$ where [x] denotes the greatest integer not exceeding x, then

$\lim\limits_{x\to 0^-} f(x) =$ **[EAMCET 2007]**

1) –1 2) 0 3) 1 4) 2

Ans: 2

Sol. $\lim_{x\to 0^-} f(x)$

$= \lim_{x\to 0^-} \frac{\sin(1+[x])}{[x]} \times \frac{1+[x]}{1+[x]}$

$= 1 \times \frac{0}{-1} = 0$

7. If 0 < p < q, then $\lim_{n\to\infty} \left(q^n + p^n\right)^{1/n} =$ **[EAMCET 2006]**

1) e 2) p 3) q 4) 0

Ans: 3

Sol. $0 < p < q,\ 0 < \frac{p}{q} < 1$

$\lim_{n\to\infty} \left(q^n + p^n\right)^{1/n} = \lim_{n\to\infty} q\left(1+\frac{p}{q}\right)^{1/n}$ = q x 1 = q

8. $\lim_{n\to\infty} \left[\sqrt{x^2+2x-1} - x\right] =$ **[EAMCET 2006]**

1) ∞ 2) 1/2 3) 4 4) 1

Ans: 4

Sol. $\lim_{x\to\infty} \frac{2x-1}{\sqrt{x^2+2x-1}+x}$

$\lim_{x\to\infty} \frac{\left(2-\frac{1}{x}\right)}{\sqrt{1+\frac{2}{x}-\frac{1}{x^2}}+1} = \frac{2}{2} = 1$

9. If $\lim_{x\to 0} \left(\frac{\cos 4x + a\cos 2x + b}{x^4}\right)$ is finite, then the values of a, b, are respectively **[EAMCET 2006]**

1) 5, –4 2) –5, –4 3) –4, 3 4) 4, 5

Ans: 3

Sol. $\lim_{x\to 0} \left(\frac{\cos 4x + a\cos 2x + b}{x^4}\right) = \lim_{x\to 0} \frac{f(x)}{g(x)}$ exists

f(0) = 0

$1 + a + b = 0 \Rightarrow a + b = -1$

$\lim_{x\to 0} \frac{f'(x)}{g'(x)} = \lim_{x\to 0} \frac{-4\sin 4x - 2a\sin 2x}{4x^3}$

$f''(0) = 0$

$-16 - 4a = 0 \Rightarrow a = -4$

$\Rightarrow b = 4 - 1 = 3$ –4, 3

10. If $I_1 = \lim_{x\to 2^+} \left(x + [x]\right), I_2 = \lim_{x\to 2^-} \left(2x + [x]\right)$ and $I_3 = \lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\left(x - \frac{\pi}{2}\right)}$, then **[EAMCET 2006]**

1) $I_1 < I_2 < I_3$ 2) $I_2 < I_3 < I_1$ 3) $I_3 < I_2 < I_1$ 4) $I_1 < I_3 < I_2$

Ans: 3

Sol. $\ell_1 = \lim\limits_{x\to 2^+} x + [x] = 4$

$\ell_2 = \lim\limits_{x\to 2^-} 2x - [x]$

$= \lim\limits_{h\to 0}\{2(2-h) - [2-h]\}$

$= 4 - 2 = 2$

$\ell_3 = (-1)\lim\limits_{\left(\frac{\pi}{2}-x\right)} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)} = -1$

$\ell_3 < \ell_2 < \ell_1$

11. $\lim\limits_{x\to 0} x^2 \sin\frac{\pi}{x} =$ **[EAMCET 2005]**

1) 1 2) 0 3) does not exist 4) ∞

Ans: 2

Sol. $\underset{x\to 0}{\text{Lt}}\, x^2 \sin\left(\frac{\pi}{x}\right) = \underset{x\to 0}{\text{Lt}}\, x^2 . \left(\underset{x\to 0}{\text{Lt}} \sin\frac{\pi}{x}\right) = 0 \times$(finite value between – 1 to 1) = 0

12. $\lim\limits_{n\to\infty} \frac{1}{n^3}\sum_{k=1}^{n}\left(k^2 x\right) =$ **[EAMCET 2004]**

1) x 2) $\frac{x}{2}$ 3) $\frac{x}{3}$ 4) $\frac{x}{4}$

Ans: 3

Sol. $x \underset{n\to\infty}{\text{Lt}} \frac{1}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right] = x \underset{n\to\infty}{\text{Lt}} \frac{2n^3 + ...}{6n^3} = x \times \frac{2}{6} = \frac{x}{3}$

13. $\lim\limits_{x\to\frac{\pi}{6}}\left(\frac{3\sin x - \sqrt{3}\cos x}{6x - \pi}\right) =$ **[EAMCET 2003]**

1) $\sqrt{3}$ 2) $\frac{1}{\sqrt{3}}$ 3) $-\frac{1}{\sqrt{3}}$ 4) $\frac{-1}{\sqrt{3}}$

Ans: 2

Sol. $\lim\limits_{x\to\frac{\pi}{6}}\left(\frac{3\sin x - \sqrt{3}\cos x}{6x - \pi}\right)$

$\lim\limits_{x\to\frac{\pi}{6}} \frac{3\cos x + \sqrt{3}\sin x}{6} = \frac{1}{\sqrt{3}}$

14. If $a > 0 \lim\limits_{x\to a} \frac{a^x - x^a}{x^x - a^a} = -1$, then a = …….. **[EAMCET 2003]**

1) 0 2) 1 3) e 4) 2e

Ans: 2

Sol. $\lim\limits_{x\to a} \frac{a^x - x^a}{x^x - a^a} = -1$

Apply L-hospital rule

$\Rightarrow \lim_{x\to a}\frac{a^x \log a - ax^{a-1}}{x^x\left(1+\log x\right)} = -1$

$\Rightarrow \frac{\log a - 1}{1+\log a} = -1 \Rightarrow \log a = 0 \Rightarrow a = 1$

15. $\lim_{x\to 0}\frac{4^x - 9^x}{x\left(4^x + 9^x\right)} =$ **[EAMCET 2002]**

1) $\log\frac{2}{3}$ 2) $\log\frac{3}{2}$ 3) $\frac{1}{2}\log\frac{2}{3}$ 4) $\frac{1}{2}\log\frac{3}{2}$

Ans: 1

Sol. $\underset{x\to 0}{\mathrm{Lt}}\frac{4^x - 9^x}{x\left(4^x + 9^x\right)}$

$\underset{x\to 0}{\mathrm{Lt}}\frac{\frac{\left(4^x - 1\right)}{x} - \frac{\left(9^x - 1\right)}{x}}{4^x + 9^x}$

$\frac{\log 4 - \log 9}{2} = \frac{1}{2}\log\left(\frac{2}{3}\right)^2 = \log\frac{2}{3}$

16. The quadratic equation whose roots are *l* and m where

$l = \lim_{\theta\to 0}\left(\frac{3\sin\theta - 4\sin^2\theta}{\theta}\right)$ and $m = \lim_{\theta\to 0}\left(\frac{2\tan\theta}{\theta\left(1-\tan^2\theta\right)}\right)$ is **[EAMCET 2002]**

1) $x^2 + 5x + 6 = 0$ 2) $x^2 - 5x + 6 = 0$ 3) $x^2 - 5x - 6 = 0$ 4) $x^2 + 5x - 6 = 0$

Ans: 2

Sol. $\ell = \underset{\theta\to 0}{\mathrm{Lt}}\left(\frac{3\sin\theta - 4\sin^2\theta}{\theta}\right)$

$= \underset{\theta\to 0}{\mathrm{Lt}}\frac{\sin\theta}{3\theta}\left(3 - 4\sin\theta\right) = 3$

$m = \underset{\theta\to 0}{\mathrm{Lt}}\frac{2\tan\theta}{\theta\left(1-\tan^2\theta\right)} = 2$

$\therefore$ The quadratic equation required is $x^2 - \left(\ell + m\right)x + \ell m = 0$

$x^2 - \left(3+2\right)x + 6 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

17. If $f: R \to R$ is defined by f(x) = x – [x], where [x] is the greatest integer not exceeding x, then the set of discontinuities of f is **[EAMCET 2002]**

1) the empty set 2) R 3) Z 4) N

Ans: 3

Sol. [x] is discontinuous at inter values of x hence
f(x) = x – [x] is discontinuous on Z

18. $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b} =$ **[EAMCET 2001]**

1) 1 2) e^{b-a} 3) e^{a-b} 4) e^{b}

Ans: 3

Sol. $e^{\underset{x\to\infty}{Lt}(x+b)\left[\frac{x-a}{x+b}-1\right]} = e^{a-b}\left[\because \underset{x\to\infty}{Lt}\ f[x]^{g(x)} = e^{\underset{x\to\infty}{Lt}\ g(x)[f(x)-1]}\right]$

19. $\lim\limits_{x\to\alpha}\frac{x.10^x - x}{1-\cos x} =$ **[EAMCET 2001]**

1) log 10 2) 2log 10 3) 3 log 10 4) 4 log 10

Ans: 2

Sol. $\lim\limits_{x\to 0}\frac{x\left(10^x-1\right)}{2\sin^2\frac{x}{2}}$

$\lim\limits_{x\to 0}\left(\frac{10^x-1}{x}\right).\underset{x\to 0}{Lt}\frac{x^2}{2\sin^2\frac{x}{2}} = 2\log 10$

20. If $f(x) = \frac{x^2-10x+25}{x^2-7x+10}$ for $x \neq 5$ and f is continuous at x = 5 then f(5) = **[EAMCET 2001]**

1) 0 2) 5 3) 10 4) 25

Ans: 1

Sol. $\lim\limits_{x\to 5}\frac{x^2-10x+25}{x^2-7x+10} = f(5)$

$\Rightarrow f(5) = 0$

21. $\lim\limits_{\theta\to\frac{\pi}{2}}\frac{1-\sin\theta}{\cos\theta\left(\frac{\pi}{2}-\theta\right)} =$ **[EAMCET 2000]**

1) 1 2) –1 3) $\frac{-1}{2}$ 4) $\frac{1}{2}$

Ans: 4

Sol. $\lim\limits_{\theta\to\frac{\pi}{2}}\frac{1-\sin\theta}{\cos\theta\left(\frac{\pi}{2}-\theta\right)}$

$$\lim_{\theta\to\frac{\pi}{2}}\frac{2\sin^2\left(\frac{\frac{\pi}{2}-\theta}{2}\right)}{2\sin\left(\frac{\frac{\pi}{2}-\theta}{2}\right)\cos\left(\frac{\frac{\pi}{2}-\theta}{2}\right).2\left(\frac{\frac{\pi}{2}-\theta}{2}\right)} = \frac{1}{2}$$

22. $\lim\limits_{x\to 0}\frac{\log_e^{(x+1)}}{3^x-1} =$ **[EAMCET 2000]**

1) $\log_e^3$ 2) 0 3) 1 4) $\log_3^e$

Ans: 4

Sol. $\lim_{x\to 0}\frac{\log_e^{(1+x)}}{3^x-1}$

$$\lim_{x\to 0}\frac{\frac{\log_e^{(1+x)}}{x}}{\left(\frac{3^x-1}{x}\right)}=\frac{\log_e^e}{\log_e^3}=\log_3^e$$

☆ ☆ ☆