

INDEFINITE INTEGRATION
PREVIOUS EAMCET BITS

1. $\int \frac{dx}{(x+1)\sqrt{4x+3}}$ [EAMCET 2009]

1) $\tan^{-1} \sqrt{4x+3} + c$ 2) $3 \tan^{-1} \sqrt{4x+3} + c$ 3) $2 \tan^{-1} \sqrt{4x+3} + c$ 4) $4 \tan^{-1} \sqrt{4x+3} + c$

Ans:

Sol: $4x+3 = t^2$ $x = \frac{t^2-3}{4}$ $= \int \frac{\frac{1}{2} t dt}{\left(\frac{t^2-3}{4} + 1\right) t}$

$4dx = 2tdt$ $= \int \frac{2}{t^2+t} dt = 2 \tan^{-1} t + c = 2 \tan^{-1} (\sqrt{4x+3}) + c$

$dx = \frac{1}{2} t dt$

2. $\int \left(\frac{2 - \sin 2x}{1 - \cos 2x} \right) e^x dx$ [EAMCET 2009]

1) $-e^x \cot x + c$ 2) $e^x \cot x + c$ 3) $2e^x \cot x + c$ 4) $-2e^x \cot x + c$

Ans: 1

Sol: $\int \left(\frac{2 - 2 \sin x \cos x}{2 \sin^2 x} \right) e^x dx = \int (\operatorname{cosec}^2 x - \cot x) e^x dx$

$= \int e^x [(-\cot x) + \operatorname{cosec}^2 x] dx$

$e^x (-\cot x) + c$ $[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c]$

3. If $I_n = \int \sin^n x dx$ then $nI_n - (n-1)I_{n-2} =$ [EAMCET 2009]

1) $\sin^{n-1} x \cos x$ 2) $\cos^{n-1} x \sin x$ 3) $-\sin^{n-1} x \cos x$ 4) $-\cos^{n-1} x \sin x$

Ans: 1

Sol: $I_n = \int \frac{\sin^{n-1} x}{u} \frac{\sin x}{v} dx$

$I_n = \sin^{n-1} x (-\cos x) - \int (n-1)(\sin x)^{n-2} (\cos x)(-\cos x) dx$

$I_n = -\sin^{n-1} x \cos x + (n-1) \int (\sin x)^{n-2} (1 - \sin^2 x) dx$

$I_n = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x dx - (n-1) \int \sin^n x dx$

$I_n = -\sin^{n-1} x \cos x + (n-1)I_{n-2} - (n-1)I_n \Rightarrow nI_n - (n-1)I_{n-2} = \sin^{n-1} x \cos x$

4. If $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx = f(x) + c$ then $f(x) =$ [EAMCET 2008]

- 1) $e^x \cot\left(\frac{x}{2}\right)$ 2) $e^{-x} \cot\left(\frac{x}{2}\right)$ 3) $-e^x \cot\left(\frac{x}{2}\right)$ 4) $-e^{-x} \cot\left(\frac{x}{2}\right)$

Ans: 3

Sol:
$$\int e^x \left(\frac{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} \right) dx = \int e^x \left[\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} - \cot \frac{x}{2} \right] dx$$

$$= e^x \left(-\cot \frac{x}{2} \right) + c \quad \left[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c \right]$$

$\therefore f(x) = -e^x \cot \frac{x}{2}$

5. If $I_n = \int x^n e^{cx} dx$ for $n \geq 1$ then $cI_n + nI_{n-1} =$ [EAMCET 2008]

- 1) $x^n e^{cx}$ 2) x^n 3) e^{cx} 4) $x^n + e^{cx}$

Ans: 1

Sol: $I_n = \int \frac{x^n}{u} \cdot \frac{e^{cx}}{v} dx$

$$x^n \int e^{cx} dx - \int n \cdot x^{n-1} \cdot \left(\frac{e^{cx}}{c} \right) dx$$

$$I_n = x^n \left(\frac{e^{cx}}{c} \right) - \frac{n}{c} I_{n-1} \Rightarrow cI_n = x^n e^{cx} - nI_{n-1}$$

$$cI_n + nI_{n-1} = x^n e^{cx}$$

6. If $\int e^x (1+x) \cdot \sec^2(xe^x) dx = f(x) + c$ then $f(x) =$ [EAMCET 2008]

- 1) $\cos(xe^x)$ 2) $\sin(xe^x)$ 3) $2 \tan^{-1} x$ 4) $\tan(xe^x)$

Ans:

Sol: Let $xe^x = t$

$$(x \cdot e^x + e^x \cdot 1) dx = dt$$

$$e^x (x+1) dx = dt$$

$$\int e^x (x+1) \sec^2(xe^x) dx = \int \sec^2 t \tan t dt$$

$$= \tan t + c \Rightarrow \tan(xe^x) + c$$

$$\therefore f(x) = \tan(xe^x)$$

7. $\int \frac{e^x - 1}{e^x + 1} dx = f(x) + c$ then $f(x)$ is equal to [EAMCET 2007]

- 1) $2\log(e^x + 1)$ 2) $\log(e^{2x} - 1)$ 3) $2\log(e^x + 1) - x$ 4) $\log(e^{2x} + 1)$

Ans: 3

$$\begin{aligned} \text{Sol: } &= \int \frac{e^x + e^x - e^x - 1}{e^x + 1} dx \Rightarrow \int \frac{2e^x - (e^x + 1)}{e^x + 1} dx \\ &= \int \frac{2e^x}{e^x + 1} dx - \int \frac{e^x + 1}{e^x + 1} dx = 2 \int \frac{e^x}{e^x + 1} dx - \int 1 dx \\ &= 2 \log|e^x + 1| - x + c \\ \therefore f(x) &= 2 \log(e^x + 1) - x \end{aligned}$$

8. $\int \tan^{-1} \left(\sqrt{\frac{1-x}{1+x}} \right) dx =$

[EAMCET 2007]

- 1) $\frac{1}{2} (x \cos^{-1} x - \sqrt{1-x^2}) + c$ 2) $\frac{1}{2} (x \cos^{-1} x + \sqrt{1-x^2}) + c$
 3) $\frac{1}{2} (x \sin^{-1} x - \sqrt{1-x^2}) + c$ 4) $\frac{1}{2} (x \sin^{-1} x + \sqrt{1-x^2}) + c$

Ans:

Sol: Put $x = \cos 2\theta$

$$\begin{aligned} &= \int \tan^{-1} \left(\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}} \right) dx \\ &= \int \tan^{-1} (\sqrt{\tan^2 \theta}) dx &&= \frac{1}{2} \left[\cos^{-1} x \cdot x - \int \frac{-1}{\sqrt{1-x^2}} \cdot x dx \right] \\ &= \int \tan^{-1} (\tan \theta) dx &&= \frac{1}{2} \left[\cos^{-1} x \cdot x - \frac{1}{2} \int \frac{-2x}{\sqrt{1-x^2}} dx \right] \\ &= \int \theta dx = \int \frac{1}{2} \cos^{-1} x dx &&= \frac{1}{2} \left[\cos^{-1} x \cdot x - \frac{1}{2} \times 2\sqrt{1-x^2} \right] + c \\ &= \frac{1}{2} \int \cos^{-1} x \cdot 1 dx &&= \frac{1}{2} \left[\cos^{-1} x \cdot x - \sqrt{1-x^2} \right] + c \end{aligned}$$

9. $\int \frac{\sin x + 8 \cos x}{4 \sin x + 6 \cos x} dx$

[EAMCET 2007]

- 1) $x + \frac{1}{2} \log|4 \sin x + 6 \cos x| + c$ 2) $2x + \log|2 \sin x + 3 \cos x| + c$
 3) $x + 2 \log|2 \sin x + 3 \cos x| + c$ 4) $\frac{1}{2} \log|4 \sin x + 6 \cos x| + c$

Ans: 1

$$\text{Sol: } \int \frac{a \cos x + b \sin x}{c \cos x + d \sin x} dx = \left(\frac{ac + bd}{c^2 + d^2} \right) x + \left(\frac{ad - bc}{c^2 + d^2} \right) \log|(\cos x + d \sin x)| + c$$

$$\int \frac{8 \cos x + 1 \cdot \sin x}{6 \cos x + 4 \sin x} dx = \left(\frac{48+4}{36+16} \right) x + \left(\frac{32-6}{36+16} \right) \log |6 \cos x + 4 \sin x| + c$$

$$= x + \frac{1}{2} \log |6 \cos x + 4 \sin x| + c$$

10. If $\int \sqrt{\frac{x}{a^3 - x^3}} dx = g(x) + c$ then $g(x)$ is equal to [EAMCET 2006]

- 1) $\frac{2}{3} \cos^{-1} x$ 2) $\frac{2}{3} \sin^{-1} \left(\frac{x^3}{a^3} \right)$ 3) $\frac{2}{3} \sin^{-1} \left(\sqrt{\frac{x^3}{a^3}} \right)$ 4) $\frac{2}{3} \cos^{-1} \left(\frac{x}{a} \right)$

Ans:

Sol: $\int \sqrt{\frac{x}{a^3 \left(1 - \frac{x^3}{a^3} \right)}} dx = \frac{1}{a} \int \frac{\sqrt{x/a}}{\sqrt{1 - \left(\frac{x}{a} \right)^3}} dx$ Let $\frac{x}{a} = \sin^{2/3} \theta$

$= \frac{1}{a} \int \frac{(\sin \theta)^{1/3}}{\sqrt{1 - \sin^2 \theta}} \times \frac{2a}{3} \times \frac{\cos \theta}{(\sin \theta)^{1/3}} d\theta$ $dx = a \cdot \frac{2}{3} (\sin \theta)^{\frac{2}{3}-1} \cos \theta d\theta$

$= \frac{2}{3} \int 1 \cdot d\theta \Rightarrow \frac{2}{3} \theta + c$ $dx = \frac{2a}{3} \times \frac{\cos \theta}{(\sin \theta)^{1/3}} d\theta$

$= \frac{2}{3} \sin^{-1} \left(\sqrt{\frac{x^3}{a^3}} \right) + c$

$\therefore g(x) = \frac{2}{3} \sin^{-1} \left(\sqrt{\frac{x^3}{a^3}} \right) + c$

11. If $\int \frac{dx}{x^2 + 2x + 2} = f(x) + c$ then $f(x)$ is equal to [EAMCET 2006]

- 1) $\tan^{-1}(x+1)$ 2) $2 \tan^{-1}(x+1)$ 3) $-\tan^{-1}(x+1)$ 4) $3 \tan^{-1}(x+1)$

Ans: 1

Sol: $\int \frac{dx}{(x+1)^2 + 1} = \tan^{-1}(x+1) + c$

$\therefore f(x) = \tan^{-1}(x+1)$

12. Observe the following statements : [EAMCET 2006]

A : $\int \left(\frac{x^2+1}{x^2} \right) e^{\frac{x^2-1}{x}} dx = e^{\frac{x^2-1}{x}} + c$

R : $\int f^1(x) \cdot e^{f(x)} dx = f(x) + c$, then which of the following is true ?

- 1) Both A and R are true and R is the correct reason of A

2) Both A and R are true and R is the correct reason of A

3) A is true, R is false

4) A is false, R is true

Ans: 3

Sol: R : $\int f^1(x) \cdot e^{f(x)} dx$

A : $\int \left(1 + \frac{1}{x^2}\right) e^{\left(x - \frac{1}{x}\right)} dx$

Let $f(x) = t$

Let $x - \frac{1}{x} = t$

$\int f^1(x) dn = dt$

$\left(1 + \frac{1}{x^2}\right) dx = dt$

$\int e^t dt = e^t + c$

$\int e^t dt = e^t + c = e^{x - \frac{1}{x}} + c$

$= e^{f(x)} + c$

$= e^{\frac{x^2-1}{x}} + c$

Here A is true and R is false

13. If $\int \frac{\sin x}{\cos x(1 + \cos x)} dx = f(x) + c$

[EAMCET 2005]

1) $\log \left| \frac{1 + \cos x}{\cos x} \right|$

2) $\log \left| \frac{\cos x}{1 + \cos x} \right|$

3) $\log \left| \frac{\sin x}{1 + \sin x} \right|$

4) $\log \left| \frac{1 + \sin x}{\sin x} \right|$

Ans: 1

Sol: Let $\cos x = t$

$-\sin x dx = dt$

$\int \frac{-dt}{t(1+t)} = \int \left(\frac{-1}{t} + \frac{1}{t+1} \right) dt$

$= -\log t + \log |t+1| + c$

$= \log \left| \frac{t+1}{t} \right| + c = \log \left| \frac{1 + \cos x}{\cos x} \right| + c$

$\therefore f(x) = \log \left| \frac{1 + \cos x}{\cos x} \right| + c$

14. $\int \frac{x^{49} \tan^{-1}(x^{50})}{1+x^{100}} dx = K \left[\tan^{-1}(x^{50}) \right]^2 + c$, then $K =$

[EAMCET 2005]

1) $\frac{1}{50}$

2) $-\frac{1}{50}$

3) $\frac{1}{100}$

4) $-\frac{1}{100}$

Ans:

Sol: Let $\tan^{-1}(x^{50}) = t$

$$\frac{1}{1+(x^{50})^2} \times 50 \cdot x^{49} dx = dt$$

$$= \frac{x^{49} dx}{1+x^{100}} = \frac{dt}{50} = \int t \frac{dt}{50} = \frac{1}{50} \times \frac{t^2}{2} + c \Rightarrow \frac{1}{100} [\tan^{-1} 50]^2 + c$$

15. If $\int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = f(x) - \log|1+x^2| + c$ then $f(x) =$ [EAMCET 2005]

- 1) $2x \tan^{-1} x$ 2) $-2x \tan^{-1} x$ 3) $x \tan^{-1} x$ 4) $-x \tan^{-1} x$

Ans: 1

Sol: $\int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = \int 2 \tan^{-1} x dx$

$$= 2 \int \tan^{-1} x \cdot 1 dx$$

$$= 2 \left[\tan^{-1} x \cdot \int 1 dx - \int \frac{1}{1+x^2} \cdot x dx \right]$$

$$= 2x \tan^{-1} x - \log|1+x^2| + c$$

$$\therefore f(x) = 2x \tan^{-1} x$$

16. $\int \frac{dx}{(x+100)\sqrt{x+99}} = f(x) + c \Rightarrow f(x) =$ [EAMCET 2004]

- 1) $2(x+100)^{1/2}$ 2) $3(x+100)^{1/2}$ 3) $2 \tan^{-1}(\sqrt{x+99})$ 4) $2 \tan^{-1}(\sqrt{x+100})$

Ans: 3

Sol: Let $x+99 = t^2$

$$dx = 2t dt$$

$$x+100 = (x+99)+1 \Rightarrow t^2+1$$

$$\int \frac{dx}{(x+100)\sqrt{x+99}} = \int \frac{2tdt}{(t^2+1)t} = 2 \tan^{-1}(t) + c$$

$$= 2 \tan^{-1}(\sqrt{x+99}) + c$$

$$\therefore f(x) = 2 \tan^{-1}(\sqrt{x+99})$$

17. $\int \frac{3-x^2}{1-2x+x^2} \cdot e^x dx = e^x f(x) + c \Rightarrow f(x) =$ [EAMCET 2004]

- 1) $\frac{1+x}{1-x}$ 2) $\frac{1-x}{1+x}$ 3) $\frac{1+x}{x-1}$ 4) $\frac{x-1}{1+x}$

Ans: 1

$$\begin{aligned}
 \text{Sol: } &= -\int \frac{x^2 - 3}{(x-1)^2} \cdot e^x dx \Rightarrow -\int \frac{(x^2 - 1) - 2}{(x-1)^2} e^x dx \\
 &= -\int \left[\frac{(x+1)(x-1)}{(x-1)^2} - \frac{2}{(x-1)^2} \right] e^x dx \\
 &= \int \left[\frac{x+1}{x-1} - \frac{2}{(x-1)^2} \right] \cdot e^x \cdot dx \Rightarrow -e^x \left[\frac{x+1}{x-1} \right] + c \\
 &= e^x \left(\frac{x+1}{1-x} \right) + c \Rightarrow f(x) = \frac{1+x}{1-x}
 \end{aligned}$$

$$18. \int \frac{\sqrt{\cot x}}{\sin x \cos x} dx = -f(x) + c \Rightarrow f(x) = \quad \text{[EAMCET 2004]}$$

- 1) $2\sqrt{\tan x}$ 2) $-2\sqrt{\tan x}$ 3) $-2\sqrt{\cot x}$ 4) $2\sqrt{\cot x}$

Ans: 4

$$\begin{aligned}
 \text{Sol: } &\int \frac{\sqrt{\cot x}}{\sin x \cos x} \times \frac{\cos \operatorname{csc}^2 x}{\cos \operatorname{csc}^2 x} dx = \int \frac{\sqrt{\cot x} \cdot \cos \operatorname{csc}^2 x}{\cot x} dx \\
 &= -\int \frac{\cos \operatorname{csc}^2 x}{\sqrt{\cot x}} dx = -2\sqrt{\cot x} + c
 \end{aligned}$$

$$\therefore f(x) = 2\sqrt{\cot x}$$

$$19. \int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx = \quad \text{[EAMCET 2003]}$$

- 1) $\frac{1}{2}\sqrt{1+x} + c$ 2) $\frac{2}{3}(1+x)^{3/2} + c$ 3) $\sqrt{1+x} + c$ 4) $2(1+x)^{3/2} + c$

Ans:

$$\begin{aligned}
 \text{Sol: } &= \int \frac{(\sqrt{1+x})^2 + \sqrt{x} \cdot \sqrt{1+x}}{\sqrt{x} + \sqrt{1+x}} dx \\
 &= \int \frac{\sqrt{1+x} [\sqrt{1+x} + \sqrt{x}]}{\sqrt{x} + \sqrt{1+x}} dx \\
 &= \int \sqrt{1+x} dx = \frac{2(1+x)^{3/2}}{3} + c
 \end{aligned}$$

$$20. \int (1+x-x^{-1})e^{x+x^{-1}} dx = \quad \text{[EAMCET 2003]}$$

- 1) $(1+x)e^{x+x^{-1}} + c$ 2) $(x-1)e^{x+x^{-1}} + c$ 3) $-xe^{x+x^{-1}} + c$ 4) $xe^{x+x^{-1}} + c$

Ans:

$$\text{use } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\int \frac{\sec^2 \frac{x}{2} dx}{12 + 2 \tan^2 \frac{x}{2}} = \frac{1}{2} \int \frac{\sec^2 \frac{x}{2} dx}{(\sqrt{6})^2 + \tan^2 \frac{x}{2}}$$

Put $\tan x/2 = t$

$$= \frac{1}{\sqrt{6}} \tan^{-1} \left(\frac{1}{\sqrt{6}} \tan \frac{x}{2} \right) + C$$

23. $\int \frac{3^x}{\sqrt{9^x - 1}} dx =$

[EAMCET 2002]

1) $\frac{1}{\log_3} \log \left| 3^x + \sqrt{9^x - 1} \right| + c$

2) $\frac{1}{\log_3} \log \left| 3^x - \sqrt{9^x - 1} \right| + c$

3) $\frac{1}{\log_9} \log \left| 3^x - \sqrt{9^x - 1} \right| + c$

4) $\frac{1}{\log_3} \log \left| 9^x + \sqrt{9^x - 1} \right| + c$

Ans: 1

Sol: $\int \frac{3^x}{\sqrt{9^x - 1}} dx$

$$3^x = t \Rightarrow 3x \log 3 dx = dt$$

$$\frac{1}{\log 3} \int \frac{dt}{\sqrt{t^2 - 1}} = \frac{1}{(\log 3)} \log \left| 3^x + \sqrt{9^x - 1} \right| + C$$

24. $\int \frac{dx}{\sqrt{x}(x+9)} =$

[EAMCET 2001]

1) $\frac{2}{3} \tan^{-1}(\sqrt{x}) + C$ 2) $\frac{2}{3} \tan^{-1}\left(\frac{\sqrt{x}}{3}\right) + C$ 3) $\tan^{-1}(\sqrt{x}) + C$ 4) $\tan^{-1}\left(\frac{\sqrt{x}}{3}\right) + C$

Ans: 2

Sol: $\int \frac{dx}{\sqrt{x}(x+9)}$

$$\text{Let } x = t^2, \quad dx = 2t dt$$

$$\Rightarrow \int \frac{2t dt}{t(t^2 + 9)}$$

$$= 2 \int \frac{dt}{t^2 + 3^2} = \frac{2}{3} \tan^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{2}{3} \tan^{-1} \left(\frac{\sqrt{x}}{3} \right) + C$$

25. $\int (x+1)^2 e^x dx =$ **[EAMCET 2001]**

- 1) $x e^x + c$ 2) $x^2 e^x + c$ 3) $(x+1) e^x + c$ 4) $(x^2+1) e^x + c$

Ans: 4

Sol: $\int (x+1)^2 e^x dx$

$$\int (x^2 + 1 + 2x) \cdot e^x dx$$

Let $f(x) = x^2 + 1 \Rightarrow f'(x) = 2x$

$$\int [f(x) + f'(x)] e^x dx = f(x) \cdot e^x + c$$

$$= (x^2 + 1) e^x + c$$

26. $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} =$ **[EAMCET 2001]**

- 1) $\frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right) + c$ 2) $\tan^{-1} \left(\frac{a \tan x}{b} \right) + c$
 3) $\frac{1}{ab} \tan^{-1} \left(\frac{b \tan x}{a} \right) + c$ 4) $\tan^{-1} \left(\frac{b \tan x}{a} \right) + c$

Ans: 1

Sol: $\int \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \int \frac{\sec^2 x dx}{a^2 \sin^2 x \sec^2 x + b^2 \cos^2 x \sec^2 x} \Rightarrow \int \frac{\sec^2 x}{a^2 \tan^2 x + b^2} dx$
 $= \frac{1}{ab} \tan^{-1} \left(\frac{a \tan x}{b} \right) + C$

27. $\int e^x (1 - \cot x + \cot^2 x) dx =$ **[EAMCET 2000]**

- 1) $e^x \cot x + c$ 2) $-e^x \cot x + c$ 3) $e^x \operatorname{cosec} + c$ 4) $-e^x \operatorname{cosec} + c$

Ans: 2

Sol: $\int e^x (1 - \cot x + \cot^2 x) dx$

$$= \int e^x (-\cot x + \operatorname{cosec}^2 x) dx$$

Let $f(x) = -\cot x \Rightarrow f'(x) = \operatorname{cosec}^2 x$

$$\Rightarrow \int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

$$= -e^x \cot x + c$$

28. $\int \frac{\sin^6 x}{\cos^8 x} dx =$

[EAMCET 2000]

1) $\tan 7x + c$

2) $\frac{\tan^7 x}{7} + c$

3) $\frac{\tan 7x}{7} + c$

4) $\sec^7 x + c$

Ans: 2

$$\text{Sol: } \int \frac{\sin^6 x}{\cos^8 x} dx = \int \tan^6 x \sec^2 x dx = \frac{\tan^7 x}{7} + c \left[\because \int f(x)^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} \right]$$

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