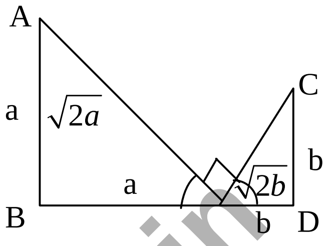
HEIGHTS AND DISTANCES

PREVIOUS EAMCET BITS

1. P is a point on the segment joining the feet of two vertical poles of heights a and b. The angles of elevation of the tops of the poles from P are 45° each. Then the square of the distance between the tops of the poles is **[EAMCET 2009]**

1) $\frac{a^2+b^2}{2}$ 2) $a^2 + b^2$ 3) $2\left(a^2+b^2\right)$ 4) $4\left(a^2+b^2\right)$

Ans: 3



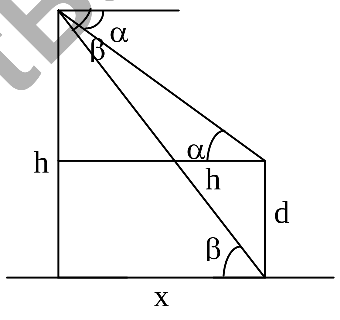
Sol. $AC^2 = \left(\sqrt{2}a\right)^2 + \left(\sqrt{2}b\right)^2$

$2\left(a^2+b^2\right)$

2. From the top of the hill h meters high the angles of depressions of the top and the bottom of a pillar are α and β respectively. The height (in metres) of the pillar is **[EAMCET 2008]**

1) $\frac{h\left(\tan\beta-\tan\alpha\right)}{\tan\beta}$ 2) $\frac{h\left(\tan\alpha-\tan\beta\right)}{\tan\alpha}$ 3) $\frac{h\left(\tan\beta+\tan\alpha\right)}{\tan\beta}$ 4) $\frac{h\left(\tan\beta+\tan\alpha\right)}{\tan\alpha}$

Ans: 1



Sol. $x = h\cot\beta, x = \left(h-d\right)\cot\alpha$

$\Rightarrow h\cot\beta = \left(h-d\right)\cot\alpha$

$\Rightarrow h\tan\alpha = \left(h-d\right)\tan\beta$

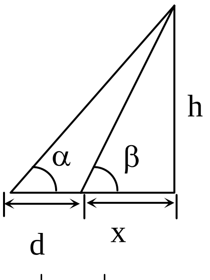
$\Rightarrow d\tan\beta = h\left(\tan\beta-\tan\alpha\right)$

$\Rightarrow d = h\frac{\left(\tan\beta-\tan\alpha\right)}{\tan\beta}$

3. The angle of elevation of an object from a point P on the level ground is α. Moving d metres on the ground towards the object, the angle of elevation is found to be β. Then the height (in metres) of the object is **[EAMCET 2007]**

1) $d\tan\alpha$ 2) $d\tan\beta$ 3) $\frac{d}{\cot\alpha+\cot\beta}$ 4) $\frac{d}{\cot\alpha-\cot\beta}$

Ans: 4



Sol. $h\cot\alpha = d + x$
$h\cot\beta = x$

$d = h\left(\cot\alpha+\cot\beta\right)$

$\therefore\ h = \frac{d}{\cot\alpha-\cot\beta}$

4. The locus of the point z = x + iy satisfying the equation $\left|\frac{z-1}{z+1}\right| = 1$ is given by **[EAMCET 2006]**

1) x = 0 2) y = 0 3) x = y 4) x + y = 0

Ans: 1

Sol. $|z-1|^2 = |z+1|^2$

$(x-1)^2 + y^2 = (x+1)^2 + y^2$

$\Rightarrow 4x = 0 \Rightarrow x = 0$

5. The product of the distinct $(2n)^{th}$ roots of $1+i\sqrt{3}$ is equal to **[EAMCET 2006]**

1) 0 2) $-1-i\sqrt{3}$ 3) $1+i\sqrt{3}$ 4) $-1+i\sqrt{3}$

Ans: 2

Sol. by substitution method put n = 1

Then $\left(1+i\sqrt{3}\right)^{\frac{1}{2}} = \left(2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right)^{1/2} = 2^{\frac{1}{2}}\left(\text{cis}\frac{\pi}{3}\right)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}\text{cis}\left(2k\pi+\frac{\pi}{3}\right)^{\frac{1}{2}}$

If k = 0, $\alpha_1 = 2^{\frac{1}{2}\text{cis}\frac{\pi}{6}}$

k = 0, $\alpha_2 = 2^{1/2}\text{cis}\left(\pi+\frac{\pi}{6}\right) = 2^{1/2}\text{cis}\frac{7\pi}{6}$

Product of roots $\alpha_1\alpha_2 = 2^{1/2}2^{1/2}\text{cis}\frac{\pi}{6}.\text{cis}\left(\frac{7\pi}{6}\right)$

$= 2\text{cis}\left(\frac{\pi}{6}+\frac{7\pi}{6}\right)$

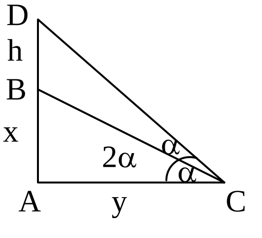
$= 2\text{cis}\frac{8\pi}{6} = 2\text{cis}\left(\frac{4\pi}{3}\right)$

$= -1-i\sqrt{3}$

6. A tower, of x meters high, has a Flagstaff at its top. The tower and the Flagstaff subtend equal angle at a point distant y metres from the foot of the tower. Then the length of the Flagstaff in metres is **[EAMCET 2005]**

1) $\frac{y\left(x^2-y^2\right)}{\left(x^2+y^2\right)}$ 2) $\frac{x\left(y^2+x^2\right)}{\left(y^2-x^2\right)}$ 3) $\frac{x\left(x^2+y^2\right)}{\left(x^2-y^2\right)}$ 4) $\frac{x\left(x^2-y^2\right)}{\left(x^2+y^2\right)}$

Ans: 2



Sol. $\tan 2\alpha = \frac{x+h}{y}, \tan\alpha = \frac{x}{y}$

Use tan(2α) formula

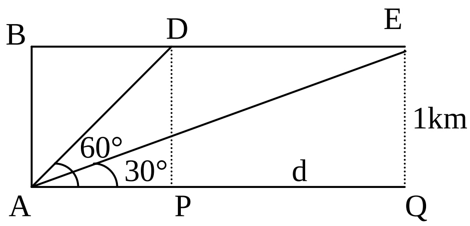
Then $h = \frac{x\left(y^2+x^2\right)}{\left(y^2-x^2\right)}$

7. An aeroplane flying with uniform speed horizontally one kilometer above the ground is observed at an elevation of 60°. After 10 seconds if the elevation is observed to be 30°, then the speed of the plane (in km/hr) is **[EAMCET 2004]**

1) $\frac{240}{\sqrt{3}}$ 2) $200\sqrt{3}$ 3) $240\sqrt{3}$ 4) $\frac{120}{\sqrt{3}}$

Ans: 3

Sol. In $\Delta APD \Rightarrow \tan 60° = \frac{1}{AP} \Rightarrow AP = \frac{1}{\sqrt{3}}$



$\Rightarrow AP + PQ = \sqrt{3}$

$PQ = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} km$

$10 \sec - \frac{2}{\sqrt{3}} km$

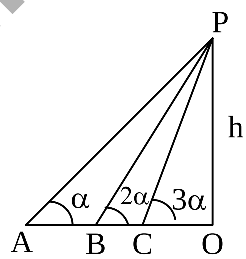
$1hr - \frac{2}{\sqrt{3}} \times \frac{3600}{10} = 240\sqrt{3} km / hr$

8. A tower subtends angles, α, 2α and 3α respectively at points A, B and C, all lying on a horizontal line through the foot of the tower. Then $\frac{AB}{BC} =$……. **[EAMCET 2003]**

1) $\frac{\sin 3\alpha}{\sin 2\alpha}$ 2) $1 + 2\cos 2\alpha$ 3) $2\cos 2\alpha$ 4) $\frac{\sin 2\alpha}{\sin \alpha}$

Ans: 2

Sol. Let height of the tower OP = h



AB = OA – OB = h(cotα - cot2α)

BC = OB – OC = h(cot2α - cot3α)

$\frac{AB}{BC} = \frac{\cot \alpha - \cot 2\alpha}{\cot 2\alpha - \cot 3\alpha}$

$= \frac{\cos \alpha \sin 2\alpha - \cos 2\alpha \sin \alpha}{\sin \alpha \sin 2\alpha} \times \frac{\sin 2\alpha . \sin 3\alpha}{\cos 2\alpha \sin 3\alpha - \cos 3\alpha \sin 2\alpha}$

$= \frac{\sin 3\alpha}{\sin \alpha} = 1 + 2\cos 2\alpha$

9. From a point on the level ground, the angle of elevation of the top of a pole is 30°. On moving 20 mts nearer, the angle of elevation is 45°. Then the height of the pole in mts is **[EAMCET 2002]**

1) $10\left(\sqrt{3} - 1\right)$ 2) $10\left(\sqrt{3} + 1\right)$ 3) 15   4) 20

Ans: 2

Sol. $\tan 30° = \frac{h}{h + 20}$

$\Rightarrow h = 10\left(\sqrt{3} + 1\right)$

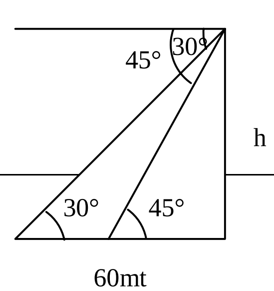
10. The shadow of the two standing on a level ground is found to be 60 metres longer when the sun's altitude is 30° then when it is 45°. The height of the tower is **[EAMCET 2001]**

1) 60 m   2) 30 m   3) 60 $\sqrt{3}$ m   4) $30\left(\sqrt{3} + 1\right)$m

Ans: 4

Sol. $h = \frac{60}{\cot 30° - \cot 45°}$

$h = \frac{60}{\sqrt{3} - 1} = 30\left(\sqrt{3} + 1\right)$



11. If two towers of height h_1 and h_2 subtend angles 60° and 30° respectively at the midpoint of the line joining their feet, then $h_1 : h_2$ = **[EAMCET 2000]**
1) 1 : 2 2) 1 : 3 3) 2 : 1 4) 3 : 1
Ans: 4

Sol. $h_1 : h_2 = \tan\alpha : \tan\beta$
$= \tan 60° : \tan 30°$
$= 3 : 1$