7. EXPONENTIAL SERIES AND LOGARITHMIC SERIES

PREVIOUS EAMCET BITS

1. $\frac{1}{e^{3x}}\left(e^{x}+e^{5x}\right)=a_0+a_1x+a_2x^2+.....\Rightarrow 2a_1+2^3a_3+2^5a_5+.......=$ **[EAMCET 2009]**

1) e 2) e^{-1} 3) 1 4) 0

Ans:

Sol: $e^{-2x}+e^{2x}$

$=2\left[1+\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}+.......\right]$

$\Rightarrow a_1=0, a_3=0, a_5=0.....$ and so on.

2. $\frac{1}{1.3}+\frac{1}{2.5}+\frac{1}{3.7}+......=$ **[EAMCET 2008]**

1) $2\log_e^2-2$ 2) $2-2\log_e^2$ 3) $2\log_e^4$ 4) $\log_e^4$

Ans: 2

Sol: $\frac{1}{1.3}+\frac{1}{2.5}+\frac{1}{3.7}+......=\sum_{n=1}^{\infty}\frac{1}{n(2n+1)}$

$=2\sum_{n=1}^{\infty}\frac{1}{2n(2n+1)}=2\sum_{n=1}^{\infty}\left[\frac{1}{2n}-\frac{1}{2n+1}\right]$

$=2\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+.....\right]$

$=2-2\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....\right]=2-2\log_e^2$

3. The coefficient of x^k in the expansion of $\frac{1-2x-x^2}{e^{-x}}$ is **[EAMCET 2007]**

1) $\frac{1-k-k^2}{k!}$ 2) $\frac{k^2+1}{k!}$ 3) $\frac{1-k}{k!}$ 4) $\frac{1}{k!}$

Ans : 1

Sol: Now $\frac{1-2x-x^2}{e^{-x}}=\left(1-2x-x^2\right)e^{x}$

$=\left[1-2x-x^2\right]\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....+\frac{x^k}{k!}+....\infty\right]$

$=\left(1+x+\frac{x^2}{2!}+....+\frac{x^k}{k!}+....\infty\right)-2\left[x+\frac{x^3}{2!}+.....+\frac{x^k}{(k-1)!}+\frac{x^{k+1}}{k!}+.....\infty\right]$

$-\left[x^2+x^3+\frac{x^4}{4!}+....+\frac{x^k}{(k-2)!}+\frac{x^{k+1}}{(k-1)!}+.....\infty\right]$

$\therefore$ coefficient of x^k is $\frac{1}{k!}-\frac{2}{(k-1)!}-\frac{1}{(k-2)!}$

$\frac{1}{k!}-\frac{2k}{k!}-\frac{k(k-1)}{k!}=\frac{1-k-k^2}{k!}$

4. $\frac{1}{2}-\frac{1}{2.2^2}+\frac{1}{3.2^3}-\frac{1}{4.2^4}+...=$ **[EAMCET 2007]**

1) $\frac{1}{4}$ 2) $\log_3\left(\frac{3}{4}\right)$ 3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{2}{3}\right)$

Ans: 2

Sol: $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+......=\log(1+x)$

$\frac{1}{2}-\frac{\left(\frac{1}{2}\right)^2}{2}+\frac{\left(\frac{1}{2}\right)^3}{3}-\frac{\left(\frac{1}{2}\right)^4}{4}+.....=\log\left(1+\frac{1}{2}\right)=\log_3\left(\frac{3}{2}\right)$

5. The coefficient of x^n in $\frac{1-2x}{e^x}$ is **[EAMCET 2006]**

1) $\frac{1+2n}{n!}$ 2) $(-1)^n\frac{(1+2n)}{n!}$ 3) $(-1)^n\frac{(1-2n)}{n!}$ 4) $(-1)^n\frac{(1+4n)}{n!}$

Ans :

Sol: $\frac{1-2x}{e^x}=(1-2x)e^{-x}$

$(1-2x)\left[1-\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........+(-1)^n\frac{x^n}{n!}+.....\right]$

Coefficient of x^n in $\frac{1-2x}{e^x}$

$=\frac{(-1)^n}{n!}-\frac{2(-1)^n}{(n-1)!}\Rightarrow(1)^n\frac{(1-2n)}{n!}$

6. If |x| < 1 and $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+......$ then x is equal to **[EAMCET 2006]**

1) $y+\frac{y^2}{2}+\frac{y^3}{3}+......$ 2) $y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+.....$

3) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$ 4) $y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+.....$

Ans:

Sol: $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.....$

$y=\log_e(1+x)\Rightarrow 1+x=e^y$

$\Rightarrow 1+x=1+y+\frac{y^2}{2!}+.....$

$\therefore x=y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$

7. $\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}=$ **[EAMCET 2005]**

1) $2e-1$ 2) $2e+1$ 3) $6e-1$ 4) $6e+1$

Ans: 3

Sol: Let $s=\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}=\sum_{n=1}^{\infty}\left[\frac{2n^2}{n!}+\frac{n}{n!}+\frac{1}{n!}\right]$

$=\sum_{n=1}^{\infty}\left[\frac{2}{(n-2)!}+\frac{3}{(n-1)!}+\frac{1}{n!}\right]$

$=2\left[1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....\infty\right]+3\left[1+\frac{1}{1!}+\frac{1}{2!}+....\infty\right]+\left[\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+...\infty\right]$

$=2e+3e+e-1=6e-1$

8. If $|a|<1, b=\sum_{n=1}^{\infty}\frac{a^k}{k}$, then a is equal to **[EAMCET 2005]**

1) $\sum_{n=1}^{\infty}\frac{(-1)^k b^k}{k}$ 2) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{k!}$ 3) $\sum_{k=1}^{\infty}\frac{(-1)^k b^k}{(k-1)!}$ 4) $\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{(k+1)!}$

Ans: 2

Sol: $b=\sum_{k=1}^{\infty}\frac{a^k}{k!}=\frac{a}{1}+\frac{a^2}{2}+\frac{a^3}{3}+....\infty$

$b=-\log_e(1-a)$

$e^{-b}=1-a\Rightarrow a=1-e^{-b}=1-\left[1-\frac{b}{1!}+\frac{b^2}{2!}-\frac{b^3}{3!}+....\infty\right]$

$\therefore a=\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{k!}$

9. The value of the series $\therefore a=\sum_{k=1}^{\infty}\frac{(-1)^{k-1} b^k}{k!}$ **[EAMCET 2004]**

1) $\cosh\left(x\log_e^a\right)$ 2) $\coth\left(x\log_e^a\right)$ 3) $\sin h\left(x\log_e^a\right)$ 4) $\tan h\left(x\log_e^a\right)$

Ans: 3.

Sol: $\frac{e^{x\log_e^a}-e^{-x\log_e^a}}{2}$ $\left[\because \frac{e^x-e^{-x}}{2}=x+\frac{x^3}{3!}+\frac{x^5}{5!}+....\right]$

$=\sinh\left[x\log_e^a\right]$

10. Coefficient of x^{10} in the expansion of $(2+3x)e^{-x}$ is **[EAMCET 2004]**

1) $-\frac{26}{(10)!}$ 2) $-\frac{28}{(10)!}$ 3) $-\frac{30}{(10)!}$ 4) $-\frac{32}{(10)!}$

Ans: 2

Sol: $(2+3x)e^{-x}=(2+3x)\left[1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}......\frac{x^9}{9!}+\frac{x^{10}}{10!}.....\right]$

$\therefore$ Coefficient of x^{10} in the above series

$=\frac{2}{10!}-\frac{3}{9!}=\frac{1}{10!}(2-30)\Rightarrow-\frac{28}{10!}$

11. $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+....$ is equal to **[EAMCET 2003]**

1) $\frac{e}{2}$ 2) $\frac{e}{3}$ 3) $\frac{e}{4}$ 4) $\frac{e}{5}$

Ans: 1

Sol: $=\sum_{n=1}^{\infty}\frac{1+2+3+.....+n}{(n+1)!}$

$=\sum_{n=1}^{\infty}\frac{n(n+1)}{2(n+1)n(n-1)!}=\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(n-1)!}$

$=\frac{1}{2}\left[1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.....\right]=\frac{1}{2}e=\frac{e}{2}$

12. If $0<y<2^{1/3}$ and $x(y^3-1)=1$, then $\frac{2}{x}+\frac{2}{3x^3}+\frac{2}{5x^5}+......$ is equal to **[EAMCET 2003]**

1) $\log\left(\frac{y^3}{2-y^3}\right)$ 2) $\log\left(\frac{y^3}{1-y^3}\right)$ 3) $\log\left(\frac{2y^3}{1-y^3}\right)$ 4) $\log\left(\frac{y^3}{1-2y^3}\right)$

Ans: 1

Sol: $y^3-1=\frac{1}{x}$

$x+\frac{x^3}{3}+\frac{x^5}{5}+......=\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$=2\left[\frac{1}{x}+\frac{\left(\frac{1}{x}\right)^3}{3}+\frac{\left(\frac{1}{x}\right)^5}{5}+.....\right]\Rightarrow 2\times\frac{1}{2}\log\left(\frac{1+1/x}{1-1/x}\right)$

$=\log\left[\frac{1+y^3-1}{1-y^3+1}\right]=\log\left[\frac{y^3}{2-y^3}\right]$

13. $1+x\log_e^a+\frac{x^2}{2!}\left(\log_e^a\right)^2+\frac{x^3}{3!}\left(\log_e^a\right)^3+......(a>0,x\in R)$ is equal to **[EAMCET 2002]**

1) a 2) a^x 3) $a^{\log_e^x}$ 4) x

Ans: 2

Sol: $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+........=e^x$

$$1+\frac{x\log_e^a}{1!}+\frac{\left(x\log_e^a\right)^2}{2!}+........=e^{x\log_e^a}=e^{\log_e^{a^x}}=a^x$$

14. $1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+.....$ is equal to **[EAMCET 2002]**

1) e^2+e 2) e^2 3) e^2-1 4) e^2-e

Ans: 4

Sol: $\sum_{n=1}^{\infty}\frac{1+2+2^2+.....+2^{n-1}}{n!}=\sum_{n=1}^{\infty}\frac{1\frac{\left(2^n-1\right)}{2-1}}{n!}$

$$=\sum_{n=1}^{\infty}\frac{2^n-1}{n!}=\sum_{n=1}^{\infty}\frac{2^n}{n!}-\sum_{n=1}^{\infty}\frac{1}{n!}$$

$$=\left(e^2-1\right)-\left(e-1\right)=e^2-e$$

15. $\frac{2}{2!}+\frac{2+4}{3!}+\frac{2+4+6}{4!}+......$ **[EAMCET 2001]**

1) e 2) e^{-1} 3) e^{-2} 4) e^{-3}

Ans: 1

Sol: $=\sum_{n=1}^{\infty}\frac{2+4+6+.........+2n}{(n+1)!}\Rightarrow\sum_{n=1}^{\infty}\frac{2(1+2+3+.....n)}{(n+1)!}$

$$=\sum_{n=1}^{\infty}\frac{n(n+1)}{(n+1)n(n-1)!}\Rightarrow\sum_{n=1}^{\infty}\frac{1}{(n-1)!}$$

$$=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.......=e$$

16. |x| < 1, the coefficient x^3 in the expansion of $\log\left(1+x+x^2\right)$ in ascending powers of x is

[EAMCET 2001]

1) $\frac{2}{3}$ 2) $\frac{4}{3}$ 3) $-\frac{2}{3}$ 4) $-\frac{4}{3}$

Ans: 3

Sol: We have $\log\left(1+x+x^2\right)=\log\left[\frac{\left(1+x+x^2\right)(1-x)}{1-x}\right]=\log\left[\frac{1-x^3}{1-x}\right]$

$$=\log\left[1-x^3\right]-\log[1-x]=-\left[x^3+\frac{x^6}{2}+\frac{x^9}{3}+......\right]+\left[x+\frac{x^2}{2}+\frac{x^3}{3}+......\right]$$

Coefficient of x^3 is $-1+\frac{1}{3}=-\frac{2}{3}$