

DIRECTION RATIOS AND DIRECTION COSINES

PREVIOUS EAMCET BITS

1. The angle between the lines whose direction cosines satisfy the equation $l + m + n = 0$, $l^2 + m^2 - n^2 = 0$ is [EAMCET 2009]

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: 3

Sol. $n = -(1+m)$

$$\therefore l^2 + m^2 - n^2 = 0 \Rightarrow l = 0 \text{ (or) } m = 0$$

$$l = 0 \Rightarrow n = -m \Rightarrow \frac{l}{0} = \frac{m}{-1} = \frac{n}{1}$$

$$m = 0 \Rightarrow n = -1 \Rightarrow \frac{l}{-1} = \frac{m}{0} = \frac{n}{1}$$

$$\therefore \cos \theta = \frac{0+0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

60. If a line in the space makes angles α , β and γ with the coordinates axes, then $\cos 2\alpha + \cos^2 \beta + \cos 2\gamma + \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$ [EAMCET 2009]

1) -1 2) 0 3) 1 4) 2

Ans: 3

Sol. $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

$$\therefore 1 - 2\sin^2 \alpha + 1 - 2\sin^2 \beta + 1 - 2\sin^2 \gamma + \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 3 - 2 = 1$$

52. The angle between the lines whose direction cosines are $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{-\sqrt{3}}{2}\right)$ is [EAMCET 2008]

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

Ans: 3

Sol. $\cos \theta = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{3}}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) = \frac{3}{16} + \frac{1}{16} - \frac{3}{4} = -\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$

52. The cosine of the angle A of the triangle with vertices A(1, -1, 2), B(6, 11, 2), C(1, 2, 6) is [EAMCET 2007]

1) $\frac{63}{65}$ 2) $\frac{36}{65}$ 3) $\frac{16}{65}$ 4) $\frac{13}{64}$

Ans: 2

Sol. $\overline{AB} = 5i + 12j; \overline{AC} = 3j + 4k$

$$\cos A = \frac{\overline{AB} \cdot \overline{AC}}{|\overline{AB}| |\overline{AC}|} = \frac{36}{65}$$

51. If the direction cosines of two lines are such that $l + m + n = 0, l^2 + m^2 - n^2 = 0$, then the angle between them is [EAMCET 2006]

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Ans: 2

Sol. $l + m + n = 0 \Rightarrow l = -(m + n)$

Substituting

$m = 0$ or $m = -n$

$l_1 : m_1 : n_1 = -1 : 0 : 1$ and $l_2 : m_2 : n_2 = 0 : -1 : 1$

$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

51. The direction cosines of the line passing through P(2, 3, -1) and the origin are [EAMCET 2005]

- 1) $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ 2) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ 3) $\frac{-2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{1}{\sqrt{14}}$ 4) $\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}$

Ans: 3

Sol. Direction cosines are $\pm \frac{x_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}},$
 $\pm \frac{y_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}, \pm \frac{z_1}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$

52. If the direction ratio of two lines are given by $l + m + n = 0, mn - 2ln + lm = 0$, then the angle between the lines is [EAMCET 2004]

- 1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) 0

Ans: 3

Sol. $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0 \Rightarrow \theta = 90^\circ$

22. If the direction ratios of two lines are given by $3lm - 4ln + mn = 0$ and $l + 2m + 3n = 0$, then the angle between the lines is [EAMCET 2003]

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

Ans: 1

Sol. $l = -(2m + 3n)$

$-3(2m + 3n)m + 4(2m + 3n)n + mn = 0$

$\Rightarrow \frac{m}{n} = \pm\sqrt{2}$

$\frac{l_1}{n_1} = -2\sqrt{2} - 3 \Rightarrow \frac{l_1}{-3 - 2\sqrt{2}} = \frac{m_1}{\sqrt{2}} = \frac{n_1}{1}$

$\frac{l_2}{-3 + 2\sqrt{2}} = \frac{m_2}{-\sqrt{2}} = \frac{n_2}{1}$

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \Rightarrow \theta = 90^\circ$$

21. The acute angle between the two lines whose direction ratios are given by $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$ is [EAMCET 2002]

- 1) 2) 3) 4)

Ans: 4

Sol. $n = l + m$ substituting in

$$l^2 + m^2 = n^2 \Rightarrow lm = 0$$

$$\text{If } l = 0 \Rightarrow m = n \Rightarrow \frac{l}{0} = \frac{m}{1} = \frac{n}{1}$$

$$m = 0 \Rightarrow l = n \Rightarrow \frac{l}{1} = \frac{m}{0} = \frac{n}{1}$$

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} = \frac{0+0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

22. The direction ratios of a normal to the plane passing through $(0, 0, 1)$ $(0, 1, 2)$ and $(1, 2, 3)$ are [EAMCET 2002]

- 1) $[0, 1, -1]$ 2) $[1, 0, -1]$ 3) $[0, 0, -1]$ 4) $[1, 0, 0]$

Ans: 1

Sol. $A(0, 0, 1); B(0, 1, 2); C(1, 2, 3)$

$$\overline{AB} = (0, 1, 1)$$

$$\overline{AC} = (1, 2, 2)$$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = (0, 1, -1)$$

21. The direction ratios of two lines are given by $a + b + c = 0$. $2ab + 2ac - bc = 0$. Then the angle between the lines is [EAMCET 2001]

- 1) π 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3}$

Ans: 2

Sol. Given $a + b + c = 0$ and $2ab + 2ac - bc = 0$

$$2a(b + c) = bc \Rightarrow -2(b + c)^2 = bc \quad (\because a = -(b + c))$$

$$\Rightarrow 2b^2 + 5bc + 2c^2 = 0$$

$$\Rightarrow (2b + c)(b + 2c) = 0$$

$$\therefore C = -2b \text{ (or) } -b/2$$

$$\text{If } C = 2b \Rightarrow a = b$$

$$\Rightarrow a : b : c = 1 : 1 : -2$$

$$\text{If } C = -b/2 \Rightarrow a = -b/2$$

$$\Rightarrow a : b : c = 1 : -2 : 1$$

If θ is the angle between the lines then $\cos \theta = \frac{(1)(1) + (1)(-2) + (-2)(1)}{\sqrt{1+1+4}\sqrt{1+4+1}} = -\frac{1}{2}$

$\therefore \theta = \frac{2\pi}{3}$

22. If a line makes angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with the x-axis and y-axis respectively, then the angle made by the line with z-axis is

[EAMCET 2001]

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{5\pi}{12}$

Ans: 2

Sol. $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2 \gamma = 1 \Rightarrow \cos^2 \gamma = \frac{1}{4} \therefore \gamma = \frac{\pi}{3}$

24. If P = (0, 1, 2), Q = (4, -2, 1), O = (0, 0, 0), then $\angle POQ =$

[EAMCET 2001]

- 1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

Ans: 4

Sol. $x_1x_2 + y_1y_2 + z_1z_2 = 0 - 2 + 2 = 0$

$\therefore \theta = 90^\circ$

24. If $\ell^2 + m^2 = 1$, then the maximum value of $\ell + m$ is

[EAMCET 2000]

- 1) 1 2) $\sqrt{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) 2

Ans: 2

Sol. Let $\ell = \cos \theta$ and $m = \sin \theta$

$\ell^2 + m^2 = 1$

The max. value of $\cos \theta + \sin \theta$ is $\sqrt{2}$

