DIFFERENTIAL EQUATIONS

PREVIOUS EAMCET BITS

1. The solution of the differential equation $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$ is **[EAMCET 2009]**

   1) $\operatorname{cosec}(x+y) + \tan(x+y) = x + c$ 2) $x + \operatorname{cosec}(x+y) = c$

   3) $x + \tan(x+y) = c$ 4) $x + \sec(x+y) = c$

   Ans: 2

Sol: Let x + y = v

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

we have, $\frac{dv}{dx} - 1 = \sin v \tan v - 1$

$\frac{dv}{dx} = \sin v \tan v$

$\frac{dv}{dx} = \frac{\sin^2 v}{\cos^2 v} \Rightarrow \int \frac{\cos v}{\sin^2 v} dt = \int dx$

$-\frac{1}{\sin v} = x + c$

$\therefore$ cosec(x + y) + x = c

2. The differential equation of the family $y = ae^x + bxe^x + cx^2e^x$ of curves, where a, b, c arbitrary constants, is **[EAMCET 2009]**

   1) $y''' + 3y'' + 3y' = 0$ 2) $y'' + 3y'' - 3y' = 0$

   3) $y''' - 3y'' - 3y' + y = 0$ 4) $y''' - 3y'' + 3y' - y = 0$

   Ans: 4

Sol: $y = ae^x + bxe^x + cx^2e^x$

$y = (a + bx + cx^2)e^x$

d.b.s.w.r.t ‘x’

$y' = (a + bx + cx^2)e^x + e^x(b + 2cx)$

$y' = y + e^x(b + 2cx)$

$y'' = y' + e^x(2cx) + (b + 2cx)e^x$

$y'' = y' + y' - y + 2cxe^x$

$y'' = 2y' - y + 2cxe^x$

$y''' = 2y'' - y' + 2ce^x$

$y''' = 2y'' - y' + \left[y'' - 2y' + y\right]$

$\therefore y''' - 3y'' + 3y' - y = 0$

3. The solution of the differential equation $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$ is **[EAMCET 2008]**

1) $(x-2y)^2 + 2x = c$ 2) $(x-2y)^2 + x = c$

3) $x - y = \log\left(\frac{cx}{y}\right)$ 4) $(x-2y) + x^2 = c$

Ans: 1

Sol: Put $x - 2y = z$.

Then $1 - 2\frac{dy}{dx} = \frac{dz}{dx} \Rightarrow 2\frac{dy}{dx} = 1 - \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(1 - \frac{dz}{dx}\right)$

$\frac{dy}{dx} = \frac{x-2y+1}{2x-4y} \Rightarrow \frac{1}{2}\left(1 - \frac{dz}{dx}\right) = \frac{z+1}{2z} \Rightarrow 1 - \frac{dz}{dx} = 1 + \frac{1}{z}$

$\Rightarrow \frac{dz}{dx} = \frac{-1}{z} \Rightarrow zdz = -dx \Rightarrow \frac{z^2}{2} = -x + c/2 \Rightarrow z^2 = -2x + c \Rightarrow (x-2y)^2 + 2x = c$

4. The solution of the differential equation $\frac{dy}{dx} - y\tan x = e^x \sec x$ is **[EAMCET 2008]**

1) $ye^x \cos x + c$ 2) $y\cos x = e^x + c$ 3) $y = e^x \sin x + c$ 4) $y\sin x = e^x + c$

Ans: 2

Sol: I.F = $e^{\int P dx} = e^{\int -\tan x dx} = \cos x$

The solution is $y\cos x = \int e^x \sec x \cos x dx = \int e^x dx = e^x + c \Rightarrow y\cos x = e^x + c$

5. The solution of the differential equation $xy^2 dy - \left(x^3 + y^3\right)dx = 0$ is **[EAMCET 2008]**

1) $y^3 = 3x^3 + c$ 2) $y^3 = 3x^3 \log(cx)$ 3) $y^3 = 3x^3 + \log(cx)$ 4) $y^3 + 3x^3 = \log(cx)$

Ans: 2

Sol: Put y = vx. Then $\frac{dy}{dx} = v + \frac{dv}{dx}$

$xy^2 dy - \left(x^3 + y^3\right)dx = 0 \Rightarrow \frac{dy}{dx} = \frac{x^3 + y^3}{xy^2} \Rightarrow v + x\frac{dv}{dx} = \frac{x^3 + x^3 v^3}{x^3 v^2} \Rightarrow v + x\frac{dv}{dx} = \frac{1}{v^2} + v$

$\Rightarrow x\frac{dv}{dx} = \frac{1}{v^2} \Rightarrow v^2 dv \Rightarrow \frac{dx}{x} = \frac{v^3}{3} = \log x + \log c$

$\Rightarrow v^3 = 3\log(cx) \Rightarrow \frac{y^3}{x^3} = 3\log(cx)$

$\therefore\ y^3 = 3x^3 \log(cx)$

6. The differential equation obtained by eliminating the arbitrary constants a and b from $xy = ae^x + be^{-x}$ is **[EAMCET 2007]**

1) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$
2) $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} - xy = 0$
3) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - y = 0$
4) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$

Ans: 1

Sol: $xy = ae^x + be^{-x}$

$\Rightarrow xy_1 + y = ae^x - be^{-x}$

$xy_2 + y_1 + y_1 = ae^x + be^{-x}$

$\therefore xy_2 + 2y_1 - xy = 0$

7. The solutions of $(x+y+1)\frac{dy}{dx} = 1$ is **[EAMCET 2007]**

1) $y = (x+2) + ce^x$ 2) $y = -(x+2) + ce^x$ 3) $x = -(y+2) + ce^y$ 4) $x = (y+2)^2 + ce^y$

Ans: 3

Sol: Put x + y + 1 = $z \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$

$(x+y+1)\frac{dy}{dx} = 1 \Rightarrow z\left(\frac{dz}{dx} - 1\right) = 1$

$\Rightarrow \frac{dz}{dx} = 1 + \frac{1}{z} \Rightarrow \int \frac{z}{z+1} dz = \int dx$

$\Rightarrow z - \log(z+1) = x + c$

$x + y + 1 = x + \log(x+y+2) + c$

$y = \log(x+y+2) + \log c$

$\Rightarrow x + y + 2 = ce^y$

8. The solution of $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is **[EAMCET 2007]**

1) $e^{y/x} = kx$ 2) $e^{y/x} = ky$ 3) $e^{-y/x} = kx$ 4) $e^{-y/x} = ky$

Ans: 2

Sol: Put y = vx

$$\frac{dy}{dx}=\frac{y^2}{xy-x^2}\Rightarrow V+x\frac{dV}{dx}=\frac{V^2}{V-1}$$

$$\Rightarrow\int\frac{V-1}{V}dV=\int\frac{1}{x}dx$$

$$\Rightarrow V-\log V=\log x+\log c$$

$$\Rightarrow V=\log\left(cVx\right)$$

$$\therefore\ cy=e^{y/x}$$

9. The solution of $\frac{dy}{dx}+1=e^{x+y}$ is **[EAMCET 2007]**

1) $e^{-(x+y)}+x+c=0$ 2) $e^{-(x+y)}-x+c=0$ 3) $e^{x+y}+x+c=0$ 4) $e^{x+y}-x+c=0$

Ans: 1

Sol: Put x + y = z $\Rightarrow 1+\frac{dy}{dx}=\frac{dz}{dx}$

$$\frac{dz}{dx}=e^z\Rightarrow\int e^{-z}dz=\int dx$$

$$-e^{-z}=x+c\Rightarrow e^{-(x+y)}+x+c=0$$

10. The solution of $\left(x^2+y^2\right)dx=2xy\,dy$ is **[EAMCET 2006]**

1) $c\left(x^2-y^2\right)=x$ 2) $c\left(x^2+y^2\right)=x$ 3) $c\left(x^2-y^2\right)=y$ 4) $c\left(x^2+y^2\right)=y$

Ans: 1

Sol: Homogeneous differential equation

Put $\frac{y}{x}=V\Rightarrow V+V'x=\frac{1+V^2}{2V}$

$$\Rightarrow\int\frac{2V}{1-V^2}dV=\int\frac{dx}{x}\ \Rightarrow-\log\left|1-v^2\right|=\log x+\log c$$

Solving $c\left(x^2-y^2\right)=x$

11. The solution of $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$ is **[EAMCET 2006]**

1) $3x\left(1+x^2\right)=4x^3+c$ 2) $3y\left(1+x^2\right)=4x^3+c$

3) $3x\left(1-x^2\right)=4x^3+c$ 4) $3y\left(1-x^2\right)=4x^3+c$

Ans: 2

Sol: $\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$

Linear differential equation $\frac{dy}{dx}+P(x)y=Q(x)$

$$I.F=e^{\int pdx}=e^{\int\frac{2x}{1+x^2}dx}=1+x^2$$

$$y.\left(1+x^2\right)=\int\frac{4x^2}{1+x^2}\left(1+x^2\right)dx$$

$$y.\left(1+x^2\right)=\frac{4x^3}{3}+c$$

$$3y\left(1+x^2\right)=4x^3+c$$

12. The solution of $\frac{dx}{dy}+\frac{x}{y}=x^2$ is **[EAMCET 2006]**

1) $\frac{1}{y}=cx-x\log x$ 2) $\frac{1}{x}=cy-y\log y$ 3) $\frac{1}{x}=cx+x\log y$ 4) $\frac{1}{y}=cx-y\log x$

Ans: 2

Sol: ------

13. $dx+dy=(x+y)(dx-dy)\Rightarrow\log(x+y)=$ **[EAMCET 2005]**

1) $x+y+c$ 2) $x+2y+c$ 3) $x-y+c$ 4) $2x+y+c$

Ans: 3

Sol: $\frac{dx+dy}{x+y}=dx-dy$

$$\int\frac{d(x+y)}{(x+y)}=\int dx-\int dy$$

$$\log(x+y)=x-y+C$$

14. $x^2y-x^3\frac{dy}{dx}=y^4\cos x\Rightarrow x^3y^{-3}=$ **[EAMCET 2005]**

1) sinx 2) $2\sin x+c$ 3) $3\sin x+c$ 4) $3\cos x+c$

Ans: 3

Sol: $x^2y-x^3\frac{dy}{dx}=y^4\cos x$

$$\frac{x^2ydx-x^3dy}{y^4dx}=\cos x$$

$$=\frac{x^2}{y^3}dx-\frac{x^3}{y^4}dx=\cos xdx$$

$$\Rightarrow\int d\left(\frac{x^3}{y^3}\right)=3\sin x+C$$

$$\int\left(\frac{3x^3}{y^3}dx-\frac{3x^3}{y^4}dy\right)=\int 3\cos x dx$$

$$\Rightarrow \frac{x^3}{y^3}=3\sin x+C$$

15. Observer the following statements : **[EAMCET 2005]**

I. $dy+2xydx=2e^{-x^2} \Rightarrow ye^{x^2}=2x+C$

II. $ye^2 2x=c \Rightarrow dx=\left|2e^{-x^2}-2xy\right|dy$

Which of the following is a correct statements

1) Both I and II are true
2) Neither I nor II is true
3) I is true , II is false
4) I is false, II is true

Ans: 3

Sol: $\frac{dy}{dx}+2x.y=2e^{-x^2}$

which is linear differential equation

$I.F=e^{\int 2xdx}=e^{x^2}$

$y.e^{x^2}=\int 2.e^{-x^2}e^{x^2}dx=2x+C$ True. $\therefore$ I is true, II is false

16. $\frac{dy}{dx}=\frac{y+x\tan y/x}{x} \Rightarrow \sin\frac{y}{x}=$ **[EAMCET 2005]**

1) cx^2
2) cx
3) cx^3
4) cx^4

Ans: 2

Sol: Put $\frac{y}{x}=V \Rightarrow \frac{dy}{dx}=V+x\frac{dv}{dx}$

$V+x.\frac{dv}{dx}=V+TanV$

$\int\frac{dv}{\tan V}=\int\frac{dv}{x}$

$\log\sin V=\log Cx$

$\sin\left(\frac{y}{x}\right)=Cx$

17. Integrating factor of $\left(x+2y^3\right)\frac{dy}{dx}=y^2$ is **[EAMCET 2004]**

1) $e^{\left(\frac{1}{y}\right)}$
2) $e^{-\left(\frac{1}{y}\right)}$
3) y
4) $\frac{-1}{y}$

Ans: 1

Sol: $\frac{dx}{dy}=\frac{x}{y^2}+2y$

$\frac{dx}{dy}+x\left(-\frac{1}{y^2}\right)=2y$

$I.F=e^{\int -\frac{1}{y^2}dy}=e^{\frac{1}{y}}$

18. $y=Ae^x+Be^{2x}+Ce^{3x}$ satisfies the differential equation **[EAMCET 2004]**

1) $y'''-6y''+11y'-6y=0$
2) $y'''+6y''+11y'-6y=0$
3) $y'''-6y''-11y'-6y=0$
4) $y'''-6y''-11y'+6y=0$

Ans: 1

Sol: $y'''-(1+2+3)y''$

$+(1.2+2.3+3.1)y'-1.2.3y=0$

19. Observe the following statements : **[EAMCET 2004]**

A : Integrating factor of $\frac{dy}{dx}+y=x^2$ is e^x

R : Integrating factor of $\frac{dy}{dx}+P(x)y=Q(x)$ is $e^{\int P(x)dx}$

Then the true statement among the following is

1) A is true, R false
2) A is false, R is true
3) A is true, R is true, $R\Rightarrow A$
4) A is false, R is false

Ans: 3

Sol: I.F of $\frac{dy}{dx}+y=x^2$ is $e^{\int 1dx}=e^x$

20. The differential equation of the family of parabola with focus at the origin and the X-axis as axis is **[EAMCET 2003]**

1) $y\left(\frac{dy}{dx}\right)^2+4x\left(\frac{dy}{dx}\right)=4y$
2) $-y\left(\frac{dy}{dx}\right)^2=2x\frac{dy}{dx}-y$
3) $y\left(\frac{dy}{dx}\right)^2+y=2xy\frac{dy}{dx}$
4) $y\left(\frac{dy}{dx}\right)^2+2xy\left(\frac{dy}{dx}\right)+y=0$

Ans: 2

Sol: Focus = (0, 0); directrix is x+ a = 0

Equation of the parabola is $y^2 = a(2x + a)$

$2y\frac{dy}{dx} = 2a \Rightarrow a = yy_1$

$y^2 = 2xyy_1 + y^2y_1^2$

$\Rightarrow y^2 = y\left[2xy_1 + yy_1^2\right]$

$\Rightarrow y = 2xy_1 + yy_1^2$

$\Rightarrow -yy_1^2 = 2xy_1 - y$

$\Rightarrow -y\left(\frac{dy}{dx}\right)^2 = 2x\left(\frac{dy}{dx}\right) - y$

21. Solution of $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$ is **[EAMCET 2003]**

1) $y\sin y = x^2 \log x + c$
2) $y\sin y = x^2 + c$
3) $y\sin y = x^2 + \log x + c$
4) $y\sin y = x\log x + c$

Ans: 1

Sol: $\frac{dy}{dx} = \frac{x\log x^2 + x}{\sin y + y\cos y}$

$\Rightarrow \sin y dy + y\cos y dy = x\log x^2 dx + x dx$

Integrating on both sides $y\sin y = x^2 \log x^2 + C$

22. The general solution of $y^2dx + \left(x^2 - xy + y^2\right)dy = 0$ is **[EAMCET 2003]**

1) $\tan^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$
2) $2\tan^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$
3) $\log\left(y + \sqrt{x^2 + y^2}\right) + \log y + c = 0$
4) $2\sinh^{-1}\left(\frac{x}{y}\right) + \log y + c = 0$

Ans: 1

Sol: $\frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$

Put y = vx

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x^2 - vx^2 + v^2x^2} = \frac{-v^2}{1 - v + v^2}$$

$$\frac{dx}{x} = -\frac{1 - v + v^2}{v\left(1 + v^2\right)}dv$$

$$\log xv = \tan^{-1} v + c$$

$$\log y = \tan^{-1}\left(\frac{y}{x}\right) + c$$

$$\Rightarrow \log y + \tan^{-1}\left(\frac{x}{y}\right) + c = 0$$

23. Order of the differential equation of the family of all concentric circles centered at (h, k) is **[EAMCET 2002]**

1) 1 2) 2 3) 3 4) 4

Ans: 1

Sol: $\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$

Centre (h, k) is fixed

Radius = r is a variable

Hence order is 1

24. The solution of $\frac{dy}{dx} + \frac{y}{3} = 1$ is **[EAMCET 2002]**

1) $y = 3 + ce^{x/3}$ 2) $y = 3 + ce^{-x/3}$ 3) $3y = c + e^{x/3}$ 4) $3y = c + c^{-x/3}$

Ans: 2

Sol: $\frac{dy}{dx} + \frac{y}{3} = 1 \Rightarrow$

Integrating factor : $I.F = e^{\int \frac{1}{3}dx} = e^{x/3}$

Solution $ye^{x/3} = \int e^{x/3}dx$

$$ye^{x/3} = 3e^{x/3} + C$$

$$y = 3 + Ce^{-x/3}$$

25. $y + x^2 = \frac{dy}{dx}$ has the solution **[EAMCET 2002]**

1) $y + x^2 + 2x + 2 = ce^x$ 2) $y + x + 2x^2 + 2 = ce^x$

3) $y + x + x^2 + 2 = ce^{2x}$ 4) $y^2 + x + x^2 + 2 = ce^{2x}$

Ans: 1

Sol: $y + x^2 = \frac{dy}{dx}$

$\frac{dy}{dx} - y = x^2$

$I.F = e^{\int -1dx} = e^{-x}$

Solution $ye^{-x} = \int x^2 e^{-x} dx$

$ye^{-x} = -e^{-x}\left(x^2 + 2x + 2\right) + C$

$y + x^2 + 2x + 2 = Ce^x$

26. The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ is **[EAMCET 2002]**

1) $x^{2/3} + y^{2/3} = c$ 2) $y^{2/3} - x^{2/3} = c$ 3) $x^{1/3} + y^{1/3} = c$ 4) $y^{1/3} - x^{1/3} = c$

Ans: 2

Sol: $\frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}} \Rightarrow y^{-1/3}dy = x^{-1/3}dx$

$\int y^{-1/3}dy = \int x^{-1/3}dx$

$\frac{3}{2}\left[y^{2/3} - x^{2/3}\right] = C_1$

$\Rightarrow y^{2/3} - x^{2/3} = C$

27. The solution of $xdx + ydy = x^2ydy - xy^2dx$ is **[EAMCET 2001]**

1) $x^2 - 1 = c\left(1 + y^2\right)$ 2) $x^2 + 1 = c\left(1 - y^2\right)$

3) $x^3 - 1 = c\left(1 + y^3\right)$ 4) $x^3 + 1 = c\left(1 - y^3\right)$

Ans: 1

Sol: $xdx + ydy = x^2ydy - xy^2dx$

$x\left(1 + y^2\right)dx = -y\left(1 - x^2\right)dy$

$\frac{-x}{1 - x^2}dx = \frac{y}{1 + y^2}dy$

$\int \frac{x}{x^2 - 1}dx = \int \frac{y}{1 + y^2}dy$

$\Rightarrow \log\left(x^2 - 1\right) = \log\left(1 + y^2\right) + \log c$

$\therefore x^2 - 1 = c\left(1 + y^2\right)$

28. The solution of $x^2 + y^2\frac{dy}{dx} = 4$ is **[EAMCET 2001]**

1) $x^2 + y^2 = 12x + c$ 2) $x^2 + y^2 = 3x + c$ 3) $x^3 + y^3 = 3x + c$ 4) $x^3 + y^3 = 12x + c$

Ans: 4

Sol: $x^2 + y^2 \frac{dy}{dx} = 4$

$\left(4 - x^2\right)dx = y^2 dy$

$\int\left(4 - x^2\right)dx = \int y^2 dy$

$x^3 + y^3 = 12x + c$

29. The solution of $\frac{dy}{dx} + y = e^x$ is **[EAMCET 2001]**

1) $2y = e^{2x} + c$ 2) $2ye^x + e^x + c$ 3) $2ye^x = e^{2x} + c$ 4) $2ye^{2x} = 2e^x + c$

Ans: 3

Sol: $\frac{dy}{dx} + y = e^x$

$P = 1; \ Q = x$

I.F is $e^{\int pdx} = e^x$

$y.e^x = \int e^x.e^x + c \Rightarrow 2ye^x = e^{2x} + c$

30. If c is a parameter, then the differential equation whose solution is $y = c^2 + \frac{c}{x}$ **[EAMCET 2000]**

1) $y = x^4\left(\frac{dy}{dx}\right) - x\left(\frac{dy}{dx}\right)^2$ 2) $y = x^4\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right)$

3) $y = x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right)$ 4) $y = x^4\left(\frac{d^2y}{dx^2}\right) - x\left(\frac{dy}{dx}\right)$

Ans: 3

Sol: $y = c^2 + \frac{C}{x} \Rightarrow \frac{dy}{dx} = \frac{-C}{x^2}$

$\Rightarrow C = -x^2 \frac{dy}{dx}$

$y = \left(-x^2 \frac{dy}{dx}\right)^2 + \frac{1}{x}\left(-x^2 \frac{dy}{dx}\right)$

$\Rightarrow \therefore y = x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right)$

31. The equation of curve passing through the origin and satisfying the differential equation $\frac{dy}{dx} = \left(x - y\right)^2$ is **[EAMCET 2000]**

1) $e^{2x}\left(1 - x + y\right) = \left(1 + x - y\right)$ 2) $e^{2x}\left(1 + x - y\right) = \left(1 - x + y\right)$

3) $e^{2x}\left(1 - x + y\right) = -\left(1 + x + y\right)$ 4) $e^{2x}\left(1 + x + y\right) = \left(1 - x + y\right)$

Ans: 1

Sol: $\frac{dy}{dx} = (x-y)^2$ Let $x - y = t$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$1 - \frac{dt}{dx} = t^2 \Rightarrow dx = \frac{dt}{1-t^2}$

$\Rightarrow \int dx = \int \frac{dt}{(1-t^2)}$

$x = \frac{1}{2}\log_e\left(\frac{1+t}{1-t}\right) \Rightarrow 2x = \log\left(\frac{1+t}{1-t}\right)$

$\Rightarrow e^{2x} = \frac{1+t}{1-t}$

$\therefore e^{2x}(1-x+y) = (1+x-y)$

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