DIFFERENTIATION
PREVIOUS EAMCET BITS

1. $x = \frac{1-\sqrt{y}}{1+\sqrt{y}} \Rightarrow \frac{dy}{dx} =$ **[EAMCET 2009]**

1) $\frac{4}{(x+1)^2}$ 2) $\frac{4(x-1)}{(1+x)^3}$ 3) $\frac{x-1}{(1+x)^3}$ 4) $\frac{4}{(1+x)^3}$

Ans: 2

Sol. Using Componendo and dividendo, then find y and $\frac{dy}{dx}$

2. $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right), y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right) \Rightarrow \frac{dy}{dx} =$ **[EAMCET 2009]**

1) 0 2) tan t 3) 1 4) sint cost

Ans: 3

Sol. $x = \tan^{-1} t, y = \tan^{-1} t$

$\Rightarrow y = x \Rightarrow \frac{dy}{dx} = 1$

3. $\frac{d}{dx}\left[a\tan^{-1} + b\log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4-1} \Rightarrow a - 2b =$ **[EAMCET 2009]**

1) 1 2) –1 3) 0 4) 2

Ans: 2

Sol. $\frac{a}{1+x^2} + \frac{b}{x-1} - \frac{b}{x+1} = \frac{1}{x^4-1}$

$\Rightarrow \frac{a}{x^2+1} + \frac{2b}{x^2-1} = \frac{1}{x^4-1}$

Put x = 0; a – 2b = 1

4. If $x = \left\{\cos\theta + \log\tan\left(\frac{\theta}{2}\right)\right\}$ and $y = a\sin\theta$ then $\frac{dy}{dx} =$ **[EAMCET 2008]**

1) cotθ 2) tanθ 3) sinθ 4) cosθ

Ans: 2

Sol. $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\cos\theta}{a\left[-\sin\theta + \frac{1}{\tan(\theta/2)} \times \sec^2\left(\frac{\theta}{2}\right).\frac{1}{2}\right]}$

$= \frac{\cos\theta}{-\sin\theta + \frac{1}{2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}} = \frac{\cos\theta}{-\sin\theta + \frac{1}{\sin\theta}} = \frac{\cos\theta\sin\theta}{1-\sin^2\theta} = \frac{\cos\theta.\sin\theta}{1-\sin^2\theta} = \tan\theta$

5. If $y=\sin(\log_e x)$ then $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=$ **[EAMCET 2008]**

1) $\sin(\log_e x)$ 2) $\cos(\log_e x)$ 3) y^2 4) $-y$

Ans: 4

Sol. $y=\sin(\log x)\Rightarrow\dfrac{dy}{dx}=\cos(\log x)\dfrac{1}{x}\Rightarrow x\dfrac{dy}{dx}=\cos(\log x)$

$\Rightarrow x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=-\sin(\log x)\dfrac{1}{x}\Rightarrow x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=-y$

6. If $2x^2-3xy+y^2+x+2y-8=0$ then $\dfrac{dy}{dx}$ **[EAMCET 2007]**

1) $\dfrac{3y-4x-1}{2y-3x+2}$ 2) $\dfrac{3y+4x+1}{2y+3x+2}$ 3) $\dfrac{3y-4x+1}{2y-3x-2}$ 4) $\dfrac{3y-4x+1}{2y+3x+2}$

Ans: 1

Sol. $\dfrac{dy}{dx}=\dfrac{-\dfrac{\partial f}{\partial x}}{\left(\dfrac{\partial f}{\partial y}\right)}=\dfrac{3y-4x-1}{2y-3x+2}$

7. If $y=\log\left\{\left(\dfrac{1+x}{1-x}\right)^{1/4}\right\}-\dfrac{1}{2}\tan^{-1}(x)$, then $\dfrac{dy}{dx}=$ **[EAMCET 2007]**

1) $\dfrac{x}{1-x^2}$ 2) $\dfrac{x^2}{1-x^4}$ 3) $\dfrac{x}{1+x^4}$ 4) $\dfrac{x}{1-x^4}$

Ans: 2

Sol. $y=\log\left(\dfrac{1+x}{1-x}\right)^{1/4}-\dfrac{1}{2}\tan^{-1}(x)$

$=\dfrac{1}{4}\left(\log(1+x)-\log(1-x)\right)-\dfrac{1}{2}\tan^{-1}x$

$\dfrac{dy}{dx}=\dfrac{1}{4}\left(\dfrac{1}{1+x}-\dfrac{1}{1-x}\right)-\dfrac{1}{2(1+x^2)}=\dfrac{x^2}{1-x^4}$

8. $x=\cos\theta, y=\sin 5\theta\Rightarrow(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}=$ **[EAMCET 2007]**

1) $-5y$ 2) $5y$ 3) $25y$ 4) $-25y$

Ans: 4

Sol. $\dfrac{dy}{dx}=\dfrac{-5\cos 5\theta}{\sin\theta}=\dfrac{-5\sqrt{1-\sin^2 5\theta}}{\sqrt{1-\cos^2\theta}}$

$y_1=-5\sqrt{\dfrac{1-y^2}{1-x^2}}$

$(1-x^2)y_1^2=25(1-y^2)\Rightarrow(1-x^2)y_2-xy_1=-25y$

9. $x^y=y^x\Rightarrow x(x-y\log x)\dfrac{dy}{dx}=$ **[EAMCET 2006]**

1) $y(y-x\log y)$ 2) $y(y+\log y)$ 3) $x(x+y\log x)$ 4) $x(y-x\log y)$

Ans: 1

Sol. $x^y = y^x$

$\Rightarrow y\log x = x\log y$

$\Rightarrow x\left(x - y\log\right)\dfrac{dy}{dx} = y\left(y - x\log y\right)$

10. If $f:\mathbb{R}\to\mathbb{R}$ is an even function which is twice differentiable on $\mathbb{R}$ and $f''(\pi) = 1$, then $f''(-\pi)$ = **[EAMCET 2005]**

1) – 1 2) 0 3) 1 4) 2

Ans: 3

Sol. Consider $f(x) = \dfrac{x^2}{2}$

$f'(x) = \dfrac{2x}{2} = x, f'(x) = 1$

$f''(\pi) = 1 = f''(-\pi)$

11. Observe the following statements : **[EAMCET 2005]**

I : $f(x) = ax^{41} + bx^{-40} \Rightarrow \dfrac{f''(x)}{f(x)} = 1640x^{-2}$

II : $\dfrac{d}{dx}\tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = \dfrac{1}{1+x^2}$

Which of the following is correct ?

1) I is true, but II is false
2) Both I and II are true
3) Neither I nor II is true
4) I is false, but II is true

Ans: 1

Sol. I) $f''(x) = 1640ax^{39} + 1640x^{-42}.b$

$= \dfrac{1640}{x^2}\left(ax^{41} + bx^{-40}\right) = \dfrac{1640}{x^2}f(x)$

$\dfrac{f''(x)}{f(x)} = 1640x^{-2}$ True

II) $\therefore\ \tan^{-1}\left(\dfrac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$\dfrac{d}{dx}\left(2\tan^{-1}x\right) = \dfrac{2}{1+x^2}$ false

12. $f(x) = 10\cos x + (13+2x)\sin x \Rightarrow f''(x) + f(x) =$ **[EAMCET 2005]**

1) cosx 2) 4cosx 3) sinx 4) 4 sinx

Ans: 2

Sol. $f'(x) = -10\sin x + (13+2x)\cos x + 2\text{snix}$

$f''(x) = -10\cos x - (13+2x)\sin x + 2\cos x + 2\cos x$

$= -f(x) + 4\cos x$

$f''(x) + f(x) = 4\cos x$

13. $x\sqrt{1+y}+y\sqrt{1+x}=0 \Rightarrow \frac{dy}{dx}=$ **[EAMCET 2005]**

1) $\frac{1}{(1+x)^2}$ 2) $\frac{-1}{(1+x)^2}$ 3) $\frac{1}{1+x^2}$ 4) $\frac{1}{1+x^2}$

Ans: 2

Sol. $x\sqrt{1+y}=-y\sqrt{1+x}$

$x^2+x^2y=y^2+y^2x$

$x^2-y^2=-xy(x-y)$

$x+y=-xy$

$y=\frac{-x}{1+x} \Rightarrow y^1=\frac{-1}{(1+x)^2}$

14. If $f:\mathbb{R}\to\mathbb{R}$ is an even function having derivatives of all orders, then an odd function among the following is **[EAMCET 2004]**

1) f'' 2) f''' 3) $f'+f''$ 4) $f''+f'''$

Ans: 2

Sol. f''' is odd, since 'f' is even

15. $x>0, x^y=e^{x-y} \Rightarrow \frac{dy}{dx}=$ **[EAMCET 2004]**

1) $\frac{1}{(1+\log x)^2}$ 2) $\frac{\log x}{(1+\log x)^2}$ 3) $\left(\frac{\log x}{1+\log x}\right)^2$ 4) $\frac{(\log x)^2}{1+\log x}$

Ans: 2

Sol. $x-y=y\log x \Rightarrow y=\frac{x}{1+\log x}$

$\Rightarrow \frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$

16. If $f(x)=\begin{cases}\frac{x-1}{3x^2-7x+5} & \text{for } x\neq 0\\ 1/3 & \text{for } x=1\end{cases}$, then $f^1(1)=$ **[EAMCET 2003]**

1) $-\frac{1}{9}$ 2) $-\frac{2}{9}$ 3) $-\frac{1}{3}$ 4) $\frac{1}{3}$

Ans: 2

Sol. $f'(1)=\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}=-2/9$

17. If $f(x)=\frac{x}{1+|x|}$ for $x\in$ then $f^1(0)=....$ **[EAMCET 2003]**

1) 0 2) 1 3) 2 4)

Ans: 2

Sol. $f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x}=1$

18. Let $f(x)=e^x, g(x)=\sin^{-1}x$ and $h(x)=f(g(x))$, then $\frac{h'(x)}{h(x)}=$ **[EAMCET 2002]**

1) $\sin^{-1}x$ 2) $\frac{1}{\sqrt{1-x^2}}$ 3) $\frac{1}{1-x^2}$ 4) $e^{\sin^{-1}x}$

Ans: 2

Sol. $h(x)=f\left[g(x)\right]=f\left(\sin^{-1}x\right)=e^{\sin^{-1}x}$

$h(x)=e^{\sin^{-1}x}\Rightarrow h^1(x)=\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

$\therefore\ \frac{h^1(x)}{h(x)}=\frac{1}{\sqrt{1-x^2}}$

19. If h(x) = e^{e^x} then $\frac{h'(x)}{h(x)}=$ **[EAMCET 2001]**

1) h(x) 2) $\frac{1}{h(x)}$ 3) log h(x) 4) –log h(x)

Ans: 3

Sol. Given $h(x)=e^{e^x}\Rightarrow \log(h(x))=e^x$

$\Rightarrow\frac{h'(x)}{h(x)}=e^x=\log h(x)$

20. If $f(x)=\frac{x^2}{x+a}$ then $f''(a)=$ **[EAMCET 2001]**

1) 4a 2) $\frac{1}{8a}$ 3) $\frac{1}{4a}$ 4) 8a

Ans: 3

Sol. $f(x)=\frac{x^2}{x+a}=x-a+\frac{a^2}{x+a}$

$f'(x)=1-\frac{a^2}{(x+a)^2}$

$f''(x)=\frac{2a^2}{(x+a)^3}$

$\therefore f''(a)=\frac{2a^2}{(a+a)^3}=\frac{1}{4a}$

21. If $y=2^{2^x}$, then $\frac{dy}{dx}=$ **[EAMCET 2000]**

1) $y(\log_{10}2)^2$ 2) $y(\log_e 2)^2$ 3) $y2^x\left(\log_e^2\right)^2$ 4) $y\log_e 2$

Ans: 3

Sol. $y=2^{2x}$

$\Rightarrow \log y=2^x\log_e^2$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2^x.\left(\log_e^2\right)^2$$

$$\therefore \frac{dy}{dx} = y.2^x.\left(\log_e^2\right)^2$$

20. $\frac{d}{dx}\left\{\cos^{-1}\left(\frac{4x^3}{27} - x\right)\right\} =$ **[EAMCET 2000]**

1) $\frac{3}{\sqrt{9-x^2}}$ 2) $\frac{1}{\sqrt{9-x^2}}$ 3) $\frac{-3}{\sqrt{9-x^2}}$ 4) $\frac{-1}{\sqrt{9-x^2}}$

Ans: 3

Sol. $y = \cos^{-1}\left(\frac{4x^3}{27} - x\right)$

$$= \cos^{-1}\left[4\left(\frac{x}{3}\right)^3 - 3\left(\frac{x}{3}\right)\right] = 3\cos^{-1}\left(\frac{x}{3}\right)$$

$$\therefore \frac{dy}{dx} = \frac{-3}{\sqrt{9-x^2}}$$