

DEFINITE INTEGRATION
PREVIOUS EAMCET BITS

1. $\int_0^{\pi} \frac{1}{1 + \sin x} dx =$

[EAMCET 2009]

- 1) 1 2) 2 3) -1 4) -2

Ans: 2

Sol: $\int_0^{\pi} \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx$
 $= \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x} dx \Rightarrow \int_0^{\pi} (\sec^2 x - \sec x \tan x) dx$
 $= (\tan x - \sec x)_0^{\pi}$
 $= [\tan \pi - \sec \pi] - [\tan 0^\circ - \sec 0^\circ]$
 $= (0 + 1) - (0 - 1) = 2$

2. $\int_0^1 x^{3/2} \sqrt{1-x} dx =$

[EAMCET 2008]

- 1) $\frac{\pi}{6}$ 2) $\frac{\pi}{9}$ 3) $\frac{\pi}{12}$ 4) $\frac{\pi}{16}$

Ans: 4

Sol: Put $x = \sin^2 \theta$ then $dx = 2 \sin \theta \cos \theta d\theta$

Also $x = 0, 1 \Rightarrow 0, \pi/2$

$\int_0^1 x^{3/2} \sqrt{1-x} dx = \int_0^{\pi/2} \sin^3 \theta \sqrt{1 - \sin^2 \theta} \cdot 2 \sin \theta \cos \theta d\theta$
 $= 2 \int_0^{\pi/2} \sin^4 \theta \cos^2 \theta d\theta = 2 \times \frac{1}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{16}$

3. $\int_{-\pi/2}^{\pi/2} \sin |x| dx =$

[EAMCET 2008]

- 1) 0 2) 1 3) 2 4) π

Ans: 3

Sol: $\int_{-\pi/2}^{\pi/2} \sin |x| dx = \int_{-\pi/2}^0 \sin(-x) dx + \int_0^{\pi/2} \sin x dx$
 $= [\cos x]_{-\pi/2}^0 + [-\cos x]_0^{\pi/2}$
 $= 1 - 0 - 0 + 1 = 2$

4. If $f(x) = \int_{-t}^t \frac{e^{-|x|}}{2} dx$ then $\lim_{t \rightarrow \infty} f(t)$ is

[EAMCET 2007]

- 1) 1 2) 1/2 3) 0 4) -1

Ans: 1

Sol: $f(t) = \int_{-t}^0 \frac{e^x}{2} dx + \int_0^t \frac{e^{-x}}{2} dx$

$$= \frac{1}{2} [e^x]_{-t}^0 + \frac{1}{2} [-e^{-x}]_0^t$$

$$= \frac{1}{2} [1 - e^{-t}] - \frac{1}{2} [e^{-t} - 1] = \frac{1}{2} - \frac{1}{2} e^{-t} - \frac{1}{2} e^{-t} + \frac{1}{2}$$

$f(t) = 1 - e^{-t}$

$\lim_{t \rightarrow \infty} f(t) = \lim_{x \rightarrow \infty} [1 - e^{-t}] = 1 - e^{-\infty} = 1 - 0 = 1$

5. $\int_0^{2\pi} \sin^6 x \cos^5 x dx =$

[EAMCET 2007]

- 1) 2π 2) $\frac{\pi}{2}$ 3) 0 4) $-\pi$

Ans: 3

Sol: $I = \int_0^{2\pi} \sin^6 x \cos^5 x dx$

Let $f(x) = \sin^6 x \cos^5 x$

$f(2\pi - x) = f(x)$

$I = 2 \int_0^{\pi} \sin^6 x \cos^5 x dx$

$f(\pi - x) = \sin^6(\pi - x) \cos^5(\pi - x)$

$= -\sin^6 x \cos^5 x = -f(x)$

$\therefore I = 0$

6. $\int_0^{\pi/2} \frac{dx}{1 + \tan^3 x} =$

[EAMCET 2006]

- 1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{3\pi}{2}$

Ans: 3

Sol: Let $I = \int_0^{\pi/2} \frac{dx}{1 + \tan^3 x}$

$$= \int_0^{\pi/2} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx \dots\dots\dots(1)$$

$$= \int_0^{\pi/2} \frac{\cos^3\left(\frac{\pi}{2} - x\right)}{\sin^3\left(\frac{\pi}{2} - x\right) + \cos^3\left(\frac{\pi}{2} - x\right)} dx$$

$$= \int_0^{\pi/2} \frac{\sin^3 x}{\cos^3 x + \sin^3 x} dx \dots\dots\dots(2)$$

(1) + (2)

$$2I = \int_0^{\pi/2} 1 dx$$

$$2I = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

7. $\int_{-1}^1 \frac{\cosh x}{1 + e^{2x}} dx =$

[EAMCET 2006]

- 1) 0 2) 1 3) $\frac{e^2 - 1}{2e}$ 4) $\frac{e^2 + 2}{2e}$

Ans: 3

Sol: $\int_{-1}^1 \frac{e^x + e^{-x}}{2(1 + e^{2x})} dx \quad \left[\because \cosh x = \frac{e^x + e^{-x}}{2} \right]$

$$= \int_{-1}^1 \frac{e^{2x} + 1}{2e^x(1 + e^{2x})} dx \Rightarrow \frac{1}{2} \int_{-1}^1 e^{-x} dx$$

$$= \frac{1}{2} \int_{-1}^1 e^{-x} dx \Rightarrow \frac{1}{2} \left[-e^{-x} \right]_{-1}^1$$

$$= -\frac{1}{2} [e^{-1} - e^1] \Rightarrow \frac{e^2 - 1}{2e}$$

8. $\int_0^{\pi/2} \frac{200 \sin x + 100 \cos x}{\sin x + \cos x} dx =$

[EAMCET 2005]

- 1) 50π 2) 25π 3) 75π 4) 150π

Ans: 3

Sol: $\int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx = (a + b) \frac{\pi}{4}$

$$\int_0^{\pi/2} \frac{200 \sin x + 100 \cos x}{\sin x + \cos x} dx = (200 + 100) \frac{\pi}{4} = 75\pi$$

9. $\int_0^{\pi} \frac{\theta \sin \theta}{1 + \cos^2 \theta} d\theta =$

[EAMCET 2005]

- 1) $\frac{\pi^2}{2}$ 2) $\frac{\pi^3}{3}$ 3) π^2 4) $\frac{\pi^2}{4}$

Ans: 4

Sol: Let $I = \int_0^{\pi} \frac{\theta \sin \theta}{1 + \cos^2 \theta} d\theta$

$$= \int_0^{\pi} \frac{(\pi - \theta) \sin(\pi - \theta)}{1 + \cos^2(\pi - \theta)} d\theta \Rightarrow \int_0^{\pi} \frac{(\pi - \theta) \sin \theta}{1 + \cos^2 \theta} d\theta$$

$$I = \pi \int_0^{\pi} \frac{\sin \theta}{1 + \cos^2 \theta} d\theta - I = 2I = \pi \cdot 2 \int_0^{\pi/2} \frac{\sin \theta}{1 + \cos^2 \theta} d\theta \text{ let } \cos \theta = t, -\sin \theta d\theta = dt$$

$$I = \pi \int_1^0 \frac{-dt}{1 + t^2} \Rightarrow \pi \int_0^1 \frac{1}{1 + t^2} dt$$

$$= \pi \left[\tan^{-1} t \right]_0^1 \Rightarrow I = \pi \left[\frac{\pi}{4} - 0 \right] \Rightarrow I = \frac{\pi^2}{4}$$

10. $\int_{-\pi/2}^{\pi/2} \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right) d\theta =$

[EAMCET 2004]

- 1) 0 2) 1 3) 2 4) -1

Ans: 1

Sol: Let $f(\theta) = \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right)$

$$f(-\theta) = \log \left(\frac{2 + \sin \theta}{2 - \sin \theta} \right) = -\log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right)$$

$$f(-\theta) = -f(\theta)$$

∴ It is an odd function

$$\therefore I = \int_{-\pi/2}^{\pi/2} \log \left(\frac{2 - \sin \theta}{2 + \sin \theta} \right) d\theta = 0$$

11. $\int_0^2 \frac{2x - 2}{2x - x^2} dx =$

[EAMCET 2004]

- 1) 0 2) 2 3) 3 4) 4

Ans: 1

Sol: Let $I = \int_0^2 \frac{2x - 2}{2x - x^2} dx$

Put $2x - x^2 = t \Rightarrow (2 - 2x)dx = dt$

$$-\int_0^0 \frac{dt}{t} = 0$$

12. $\int_{-2}^2 [x] dx =$ [EAMCET 2003]

- 1) 1 2) 2 3) 3 4) 4

Ans: 4

Sol: $\int_{-2}^{-1} [x] dx + \int_{-1}^0 [x] dx + \int_0^1 [x] dx + \int_1^2 [x] dx$

$$= 2 + 1 + 0 + 1 = 4$$

13. $\int_0^1 \sin\left(2 \tan^{-1} \sqrt{\frac{1+x}{1-x}}\right) dx =$ [EAMCET 2003]

- 1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

Ans: 2

Sol: Put $x = \cos\theta$ L.L : $0 = \cos\theta \Rightarrow \theta = \pi/2$
 $dx = -\sin\theta d\theta$ U.L : $1 = \cos\theta \Rightarrow \theta = 0^\circ$

$$= \int_{\pi/2}^0 \sin\left[2 \tan^{-1} \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\right] (-\sin\theta) d\theta$$

$$= \int_0^{\pi/2} \sin(\pi - \theta) \sin\theta d\theta = \int_0^{\pi/2} \sin^2 \theta d\theta$$

$$= \frac{1}{2} \times \frac{\pi}{2} = \frac{\pi}{4}$$

14. $\int_0^3 \frac{3x+1}{x^2+9} dx =$ [EAMCET 2003]

- 1) $\text{Log}(2\sqrt{2}) + \frac{\pi}{12}$ 2) $\text{Log}(2\sqrt{2}) + \frac{\pi}{2}$ 3) $\text{Log}(2\sqrt{2}) + \frac{\pi}{6}$ 4) $\text{Log}(2\sqrt{2}) + \frac{\pi}{3}$

Ans: 1

Sol: $= \frac{3}{2} \int_0^3 \frac{2x}{x^2+9} dx + \int_0^3 \frac{1}{x^2+9} dx$

$$= \frac{3}{2} [\text{Log}(x^2+9)]_0^3 + \frac{1}{3} \left[\tan^{-1} \frac{x}{3} \right]_0^3$$

$$= \frac{3}{2} [\text{Log}18 - \text{Log}9] + \frac{1}{3} [\tan^{-1}(1) - \tan^{-1}(0)]$$

$$= \frac{3}{2} \text{Log}2 + \frac{\pi}{12} = \text{Log}2^{3/2} + \frac{\pi}{12}$$

$$= \text{Log}(2\sqrt{2}) + \frac{\pi}{12}$$

15. $\int_2^3 \frac{dx}{x^2 - x} =$ [EAMCET 2002]

- 1) $\log \frac{2}{3}$ 2) $\log \frac{4}{3}$ 3) $\log \frac{8}{3}$ 4) $\log \frac{1}{4}$

Ans: 2

Sol: $\int_2^3 \frac{dx}{x^2 - x} = \int_2^3 \frac{dx}{x(x-1)}$

$$= \int_2^3 \left(\frac{1}{x-1} - \frac{1}{x} \right) dx = \left[\log \left(\frac{x-1}{x} \right) \right]_{x=2}^3$$

$$= \log \frac{2}{3} - \log \frac{1}{2} = \log \frac{4}{3}$$

16. $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx =$ [EAMCET 2002]

- 1) $\frac{3\pi}{128}$ 2) $\frac{3\pi}{256}$ 3) $\frac{3\pi}{572}$ 4) $\frac{3\pi}{64}$

Ans: 2

Sol: $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x dx = 2 \int_0^{\pi/2} \sin^4 x \cos^6 x dx$

$$= 2 \left[\frac{3}{10} \cdot \frac{1}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} \right] = \frac{3\pi}{256}$$

17. $\int_0^{\pi/2} \sin^8 x \cos^2 x dx =$ [EAMCET 2001]

- 1) $\frac{\pi}{512}$ 2) $\frac{3\pi}{512}$ 3) $\frac{5\pi}{512}$ 4) $\frac{7\pi}{512}$

Ans: 4

Sol: $\int_0^{\pi/2} \sin^8 x \cos^2 x dx$

$$= \frac{7}{10} \cdot \frac{5}{8} \cdot \frac{3}{6} \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{7\pi}{512}$$

18. $\int_{-1}^1 (ax^3 + bx) dx = 0$ for ; [EAMCET 2001]

- 1) any values of a and b
3) $a > 0$ and $b < 0$ only

- 2) $a > 0$ and $b > 0$ only
4) $a < 0$ and $b < 0$ only

Ans: 1

Sol: $ax^3 + bx$ is odd function

$$\therefore \int_{-1}^1 (ax^3 + bx) dx = 0$$

for any values of 'a' and 'b'

19. $\left(\sum_{n=1}^{10} \int_{2n-1}^{-2n} \sin^{27} x dx \right) + \left(\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x dx \right)$

[EAMCET 2000]

- 1) 27^2 2) -54 3) 54 4) 0

Ans: 4

Sol: $\sin^{27} x$ is odd function

20. $\int_0^1 \frac{x}{(1-x)^{5/4}} dx =$

[EAMCET 2000]

- 1) $\frac{16}{3}$ 2) $\frac{3}{16}$ 3) $-\frac{3}{16}$ 4) $-\frac{16}{3}$

Ans: 4

Sol: $\int_0^1 \frac{x}{(1-x)^{5/4}} dx = \int_0^1 \frac{1-x}{[(1-1-x)^{5/4}]} dx \Rightarrow \int_0^1 \frac{1-x}{x^{5/4}} dx$
 $= \int_0^1 \left(\frac{1}{x^{5/4}} - \frac{1}{x^{1/4}} \right) dx = \frac{-16}{3}$

21. If $f(x)$ is integrable on $[0, a]$ then $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx =$

[EAMCET 2000]

- 1) 0 2) 1 3) a 4) $a/2$

Ans: 4

Sol: $I = \int_0^a \frac{f(x)}{f(x)+f(a-x)} dx \dots\dots\dots(1)$

$\Rightarrow I = \int_0^a \frac{f(a-x)}{f(a-x)+f(x)} dx \dots\dots\dots(2)$

(1) + (2)

$\Rightarrow 2I = \int_0^a I dx \Rightarrow I = \frac{a}{2}$

$$22. \lim_{n \rightarrow \infty} \left[\frac{1}{2n+1} + \frac{1}{2n+2} + \dots + \frac{1}{2n+n} \right]$$

[EAMCET 2000]

$$1) \log_e \left(\frac{1}{3} \right) \quad 2) \log_e \left(\frac{2}{3} \right) \quad 3) \log_e \left(\frac{3}{2} \right) \quad 4) \log_e \left(\frac{4}{3} \right)$$

Ans: 3

$$\text{Sol: } \lim_{n \rightarrow \infty} \left[\frac{1}{2n+1} + \frac{1}{2n+2} + \dots + \frac{1}{2n+n} \right]$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \left(\frac{1}{2n+r} \right)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{n \left(2 + \frac{r}{n} \right)}$$

$$= \int_0^1 \frac{dx}{2+x} = \left[\log_e (2+x) \right]_0^1 = \log_e \left(\frac{3}{2} \right)$$

