DEMOVIRE'S THEOREM AND TRIGONOMETRIC EXPANSION

PREVIOUS EAMCET BITS

1. A value of a such that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^n = 1$ is **[EAMCET 2007]**

1) 12 2) 3 3) 2 4) 1

Ans: 1

Sol. $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^n = 1 \Rightarrow \operatorname{cis}\frac{n\pi}{6} = 1$

$\therefore$ n = 12

2. If α is a non-real root of $x^6 = 1$, then $\frac{\alpha^5+\alpha^3+\alpha+1}{\alpha^2+1} =$ **[EAMCET 2005]**

1) α^2 2) 0 3) $-\alpha^2$ 4) α

Ans: 3

Sol. $x^6 = 1 \Rightarrow \text{let } \alpha = \omega$

$\frac{\alpha^5+\alpha^3+\alpha+1}{\alpha^2+1} = \frac{\omega^5+\omega^3+\omega+1}{\omega^2+1}$

$= -\omega^2 = -\alpha^2$

3. If $x_n = \cos\frac{\pi}{2^n} + i\sin\frac{\pi}{2^n}$, then $\prod_{n=1}^{\infty} x_n =$ **[EAMCET 2004]**

1) -1 2) 1 3) $\frac{1}{\sqrt{2}}$ 4) $\frac{i}{\sqrt{2}}$

Ans: 1

Sol. $x_n = \operatorname{cis}\frac{\pi}{2^n} \Rightarrow x_1x_2x_3.....$

$= \operatorname{cis}\frac{(\pi/2)}{1-\frac{1}{2}} = \operatorname{cis}\pi = -1$

4. If $\sin 6\theta = 32\cos^2\theta\sin\theta - 32\cos^3\theta\sin\theta + 3x$, then $x = ...$ **[EAMCET 2003]**

1) $\cos\theta$ 2) $\cos 2\theta$ 3) $\sin\theta$ 4) $\sin 2\theta$

Ans: 4

Sol. $\sin 6\theta = 2\sin 3\theta\cos 3\theta$

$= 2\left(3\sin\theta - 4\sin^3\theta\right)\left(4\cos^3\theta - 3\cos\theta\right)$

$= 32\cos^5\theta\sin\theta - 32\cos^3\theta\sin\theta + 3\sin 2\theta$

$\therefore$ x = sin2θ

5. If $x_n = \cos\left(\frac{\pi}{4^n}\right) + i\sin\left(\frac{\pi}{4^n}\right)$, $x_1x_2x_3........\infty =$ **[EAMCET 2002]**

1) $\frac{1+i\sqrt{3}}{2}$ 2) $\frac{-1+i\sqrt{3}}{2}$ 3) $\frac{1-i\sqrt{3}}{2}$ 4) $\frac{-1-i\sqrt{3}}{2}$

Ans: 1

Sol. $x_1 x_2 ....\infty$

$$= \text{cis}\frac{\pi}{4}.\text{cis}\frac{\pi}{4^2}.\text{cis}\frac{\pi}{4^3}......\infty$$

$$= \text{cis}\left(\frac{\pi}{4}+\frac{\pi}{4^2}+......\infty\right) = \text{cis}\left(\frac{\pi/4}{1-1/4}\right)$$

$$= \text{cis}\frac{\pi}{3} = \frac{1+i\sqrt{3}}{2}$$

6 If θ = π/6, then the tenth term of $1+(\cos\theta+i\sin\theta)+(\cos\theta+i\sin\theta)^2+...$ is **[EAMCET 2001]**

1) i 2) – 1 3) 1 4) –1

Ans: 4

Sol. $1+\text{cis}\,\theta+(\text{cis}\,\theta)^2+......,\text{10th term} = \text{cis}9\theta$

$$= \cos 9\theta + i\sin 9\theta$$

at $\theta = \frac{\pi}{6} \Rightarrow \text{10th term} = -i$

7 $\frac{\sin 5\theta}{\sin\theta} =$ **[EAMCET 2001]**

1) $16\cos^4\theta - 12\cos^2\theta + 1$ 2) $16\cos^4\theta + 12\cos^2\theta + 1$

3) $16\cos^4\theta - 12\cos^2\theta - 1$ 4) $16\cos^4\theta + 12\cos^2\theta - 1$

Ans: 1

Sol. $\sin 5\theta = 16\cos^4\theta.\sin\theta - 12\cos^2\theta.\sin\theta + \sin\theta$

$$\therefore \frac{\sin 5\theta}{\sin\theta} = 16\cos^4\theta - 12\cos^2\theta + 1$$

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