

**CONTINUITY**  
**PREVIOUS EAMCET BITS**

1. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = \begin{cases} \frac{2 \sin x - \sin 2x}{2x \cos x} & \text{if } (x \neq 0) \\ a & \text{if } (x = 0) \end{cases}$

Then the value of  $a$  so that  $f$  is continuous at  $0$  is

[EAMCET 2009]

- 1) 2                      2) 1                      3) -1                      4) 0

Ans: 4

2. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2} & \text{for } x \neq 0 \\ \lambda & \text{for } x = 0 \end{cases}$  and if  $f$  is continuous at  $x = 0$ ,

then  $\lambda =$

[EAMCET 2008]

- 1) -2                      2) -4                      3) -6                      4) -8

Ans: 2

Sol.  $f$  is continuous at

$$x = 0 \Rightarrow \lim_{x \rightarrow 0} f(x) = f(0)$$

$$\Rightarrow \lambda = \lim_{x \rightarrow 0} \frac{\cos 3x - \cos x}{x^2} = \lim_{x \rightarrow 0} \frac{-3 \sin 3x + \sin x}{2x}$$

$$= \lim_{x \rightarrow 0} \frac{-9 \cos 3x + \cos x}{2} = \frac{-9 + 1}{2} = -4$$

3. If  $f(x) = \begin{cases} x-5 & \text{for } x \leq 1 \\ 4x^2 - 9 & \text{for } 1 < x < 2 \\ 3x + 4 & \text{for } x \geq 2 \end{cases}$  then  $f(2^+) - f(2^-) =$

[EAMCET 2007]

- 1) 0                      2) 2                      3) 3                      4) 4

Ans: 3

Sol.  $f(2^+) - f(2^-)$

$$= (3(2) + 4) - (4(2)^2 - 9) = 3$$

4. If  $f(x) = \begin{cases} \frac{1 - \sqrt{2} \sin x}{\pi - 4x} & \text{if } x \neq \frac{\pi}{4} \\ a & \text{if } x = \frac{\pi}{4} \end{cases}$  is continuous at  $\frac{\pi}{4}$  then  $a =$

[EAMCET 2006]

- 1) 4                      2) 2                      3) 1                      4) 1/4

Ans: 4

Sol.  $f(x)$  is continuous at  $x = \frac{\pi}{4}$

$$\lim_{x \rightarrow \frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$$

$$a = \lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \sqrt{2} \sin x}{\pi - 4x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{f'(x)}{g'(x)}$$

$$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-\sqrt{2} \cos x}{-4} = \frac{\sqrt{2}}{4} \cdot \frac{1}{\sqrt{2}} = \frac{1}{4}$$

5. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = \begin{cases} \frac{x-2}{x^2-3x+2} & \text{if } x \in \mathbb{R} - (1,2) \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x = 2 \end{cases}$  then  $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} =$

[EAMCET 2005]

- 1) 0                      2) -1                      3) 1                      4) -1/2  
 Ans: 2

Sol.  $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{\frac{x-2}{x^2-3x+2} - 1}{x - 2}$

$$= \lim_{x \rightarrow 2} \frac{\frac{1}{x-2} - 1}{x - 2} = \lim_{x \rightarrow 2} \frac{-(x-2)}{x - 2} = -1$$

6. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = \begin{cases} \frac{x+2}{x^2+3x+2} & \text{if } x \in \mathbb{R} - \{-1, -2\} \\ -1 & \text{if } x = -2 \\ 0 & \text{if } x = -1 \end{cases}$  then  $f$  is continuous on the set

[EAMCET 2005]

- 1)  $\mathbb{R}$                       2)  $\mathbb{R} - \{-2\}$                       3)  $\mathbb{R} - \{-1\}$                       4)  $\mathbb{R} - \{-1, -2\}$   
 Ans: 3

Sol.  $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{x+2}{x^2+3x+2} =$

$$\lim_{x \rightarrow -1^-} \frac{1}{x+1} = -\infty, \quad \lim_{x \rightarrow -1^+} f(x) = \infty$$

$\therefore \lim_{x \rightarrow -1^-} \neq \lim_{x \rightarrow -1^+}$

$\therefore f(x)$  is not continuous at  $x = -1$

7. If  $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{for } -1 \leq x < 0 \\ 2x^2 + 3x - 2 & \text{for } 0 \leq x \leq 1 \end{cases}$  is continuous at  $x = 0$ , then  $k = \dots$  [EAMCET 2003]

- 1) -4                      2) -3                      3) -2                      4) -1  
 Ans: 3

Sol.  $\lim_{x \rightarrow 0} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}$

$$= \lim_{x \rightarrow 0} (2x^2 + 3x - 2)$$

$$= \lim_{x \rightarrow 0} \frac{2k}{\sqrt{1+kx} + \sqrt{1-kx}} = -2$$

$$k = -2$$

8. If  $f : |\mathbb{R} \rightarrow \mathbb{R}|$  defined by  $f(x) = \begin{cases} a^2 \cos^2 x + b^2 \sin^2 x & (x \leq 0) \\ e^{ax+b} & (x > 0) \end{cases}$  is a continuous function, then

[EAMCET 2002]

- 1)  $b = 2 \log|a|$       2)  $2b = \log|a|$       3)  $b = \log|2a|$       4)  $b^2 = \log|a|$

Ans: 1

Sol.  $\lim_{x \rightarrow 0} f(x) = f(0)$   
 $= \lim_{x \rightarrow 0^+} e^{ax+b} = a^2(1) + b^2(0)$   
 $e^b = a^2 \Rightarrow b = 2 \log|a|$

9. If  $f(x) = \begin{cases} 2x + b & (x < \alpha) \\ x + d & (x \geq \alpha) \end{cases}$  is such that  $\lim_{x \rightarrow \alpha} f(x) = \ell$ , then  $\ell =$  [EAMCET 2001]

- 1)  $2d - b$       2)  $2b - d$       3)  $2d + b$       4)  $b - 2d$

Ans: 1

Sol.  $\lim_{x \rightarrow \alpha^-} f(x) = \lim_{x \rightarrow \alpha^+} f(x)$   
 $\lim_{x \rightarrow \alpha^-} (2x + b) = \lim_{x \rightarrow \alpha^+} (x + d)$   
 $2\alpha + b = \alpha + d \Rightarrow \alpha = d - b$   
 $\therefore \lim_{x \rightarrow \alpha} (2x + b) = \ell \Rightarrow 2\alpha + b = \ell$   
 $\Rightarrow 2(d - b) + b = \ell$   
 $\therefore \ell = 2d - b$

10. If the function  $f(x) = \begin{cases} \frac{\sin 3x}{x} & (x \neq 0) \\ \frac{k}{2} & (x = 0) \end{cases}$  is continuous at  $x = 0$  then  $k =$  [EAMCET 2000]

- 1) 3      2) 6      3) 9      4) 12

Ans: 2

Sol.  $\lim_{x \rightarrow 0} \left( \frac{\sin 3x}{x} \right) = \frac{k}{2} \Rightarrow 3 = \frac{k}{2}$   
 $\Rightarrow \therefore k = 6$

