COMPLEX NUMBERS

PREVIOUS EAMCET BITS

1. The locus of z satisfying the inequality $\left|\frac{z+2i}{2z+i}\right| < 1$, where z = x + iy, is **[EAMCET 2009]**

   1) $x^2+y^2<1$ 2) $x^2-y^2<1$ 3) $x^2+y^2>1$ 4) $2x^2+3y^2<1$

   Ans: 3

Sol. $|x+i(y+2)|^2 < |2x+i(2y+1)|^2$

$\Rightarrow x^2+y^2>1$

2. The points in the set $\left\{z\in C : \text{Arg}\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2}\right\}$ lie on the curve which is a (where C denotes the set of all complex numbers) **[EAMCET 2008]**

   1) circle 2) pair of lines 3) parabola 4) hyperbola

   Ans: 1

Sol. $\frac{z-2}{z-6i} = \frac{(x-2)+iy}{x+i(y-6)} = \frac{[(x-2)+iy][x-i(y-6)]}{x^2-(y-6)^2}$

$= \frac{x(x-2)+y(y-6)}{x^2+(y-6)^2} + \frac{xy-(x-2)(y-6)}{x^2+(y-6)^2}i$

$\text{Arg}\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2} \Rightarrow \tan^{-1}\frac{xy-(x-2)(y-6)}{[x(x-2)+y(y-6)]}=\frac{\pi}{2}$

$\Rightarrow \frac{xy-(x-2)(y-6)}{x(x-2)+y(y-6)}=\frac{1}{0}$

$\Rightarrow x(x-2)+y(y-6)=0 \Rightarrow x^2+y^2-2x-6y=0 \Rightarrow (x,y)$ lies on a circle.

3. If ω is a complex cube root of unity, then $\sin\left\{(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right\}$ = **[EAMCET 2008]**

   1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{2}$ 3) 1 4) $\frac{\sqrt{3}}{2}$

   Ans: 1

Sol. $\sin\left[(\omega^{10}+\omega^{23})\pi-\frac{\pi}{4}\right]=\sin\left[(\omega+\omega^2)\pi-\frac{\pi}{4}\right]=\sin\left(-\pi-\frac{\pi}{4}\right)=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

4. If m_1, m_2, m_3 and m_4 respectively denote the moduli of the complex numbers 1 + 4i, 3+i, 1-i and 2-3i, then the correct one, among the following is **[EAMCET 2008]**

   1) $m_1<m_2<m_3<m_4$ 2) $m_4<m_3<m_2<m_1$

   3) $m_3<m_2<m_4<m_1$ 4) $m_3<m_1<m_2<m_4$

   Ans: 3

Sol. $m_1=|1+4i|=\sqrt{*+16}=\sqrt{17}$, $m_2=|3+i|=\sqrt{9+1}=\sqrt{10}$

$m_3=|1-i|=\sqrt{1+1}=\sqrt{2}, m_4=|2-3i|=\sqrt{4+9}=\sqrt{13}$

$\therefore\ m_3 < m_2 < m_4 < m_1$

5. If $a = \dfrac{1 - i\sqrt{3}}{2}$ then the correct matching of List – I from List – II is **[EAMCET 2007]**

List – I	List – II
i) $a\bar{a}$	A) $\dfrac{2\pi}{3}$
ii) $\arg\left(\dfrac{1}{a}\right)$	B) $-i\sqrt{3}$
iii) $a - \bar{a}$	C) $2i/\sqrt{3}$
iv) $\operatorname{Im}\left(\dfrac{4}{3a}\right)$	D) 1
	E) $\pi/3$
	F) $\dfrac{2}{\sqrt{3}}$

	i	ii	iii	iv		i	ii	iii	iv
1)	D	E	C	B	2)	D	A	B	F
3)	F	E	B	C	4)	D	A	B	C

Ans: 2

Sol. i) $a.\bar{a} = \left(\dfrac{1 - i\sqrt{3}}{2}\right)\left(\dfrac{1 + i\sqrt{3}}{2}\right) = 1 = D$

ii) $\operatorname{Arg}\left(\dfrac{1}{\bar{a}}\right) = \operatorname{Arg}\left(\dfrac{1 - i\sqrt{3}}{2}\right) = \dfrac{2\pi}{3} = A$

iii) $a - \bar{a} = -i\sqrt{3} = B$

iv) $\operatorname{Im}\left(\dfrac{4}{3a}\right) = \operatorname{Im}\left(\dfrac{8}{3\left(1 - \sqrt{3}i\right)}\right) = \dfrac{2}{\sqrt{3}} = F$

6. The locus of the point z = x + iy satisfying $\left|\dfrac{z - 2i}{z + 2i}\right| = 1$ is **[EAMCET 2007]**

1) x-axis 2) y-axis 3) y = 2 4) x = 2

Ans: 1

Sol. $\left|Z - 2i\right| = \left|Z + 2i\right|$

$x^2 + \left(y - 2\right)^2 = x^2 + \left(y + 2\right)^2 \Rightarrow y = 0$

$\therefore$ Locus is x-axis

7. The locus of the point z = x + iy satisfying the equation $\left|\dfrac{z - 1}{z + 1}\right| = 1$ is given by **[EAMCET 2006]**

1) x = 0 2) y = 0 3) x = y 4) x + y = 0

Ans: 1

Sol. $\left|z - 1\right|^2 = \left|z + 1\right|^2$

$\left(x - 1\right)^2 + y^2 = \left(x + 1\right)^2 + y^2$

$\Rightarrow 4x = 0 \Rightarrow x = 0$

8. The product of the distinct $(2n)^{th}$ roots of $1+i\sqrt{3}$ is equal to **[EAMCET 2006]**

1) 0 2) $-1-i\sqrt{3}$ 3) $1+i\sqrt{3}$ 4) $-1+i\sqrt{3}$

Ans: 2

Sol. by substitution method put n = 1

Then $\left(1+i\sqrt{3}\right)^{\frac{1}{2}} = \left(2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right)^{1/2} = 2^{\frac{1}{2}}\left(\text{cis}\frac{\pi}{3}\right)^{\frac{1}{2}}$

$= 2^{\frac{1}{2}}\text{cis}\left(2k\pi+\frac{\pi}{3}\right)^{\frac{1}{2}}$

If k = 0, $\alpha_1 = 2^{\frac{1}{2}\text{cis}\frac{\pi}{6}}$

k = 0, $\alpha_2 = 2^{1/2}\text{cis}\left(\pi+\frac{\pi}{6}\right) = 2^{1/2}\text{cis}\frac{7\pi}{6}$

Product of roots $\alpha_1\alpha_2 = 2^{1/2}2^{1/2}\text{cis}\frac{\pi}{6}.\text{cis}\left(\frac{7\pi}{6}\right)$

$= 2\text{cis}\left(\frac{\pi}{6}+\frac{7\pi}{6}\right)$

$= 2\text{cis}\frac{8\pi}{6} = 2\text{cis}\left(\frac{4\pi}{3}\right)$

$= -1-i\sqrt{3}$

9. If $\alpha_1, \alpha_2, \alpha_3$ respectively denote the moduli of the complex number – i, $\frac{1}{3}(1+i)$ and –1 + i, then their increasing order is **[EAMCET 2005]**

1) $\alpha_1, \alpha_2, \alpha_3$ 2) $\alpha_3, \alpha_2, \alpha_1$ 3) $\alpha_2, \alpha_1, \alpha_3$ 4) $\alpha_3, \alpha_1, \alpha_2$

Ans: 3

Sol. $\alpha_1 = |-i| = 1, \frac{1}{3}|1+i| = \frac{\sqrt{2}}{3} = \alpha_2, |-1+i| = \sqrt{2} = \alpha_3$

$\alpha_2, \alpha_1, \alpha_3$

10. If z_1, z_2 are two complex numbers satisfying $\left|\frac{z_1-3z_2}{3-z_1\bar{z}_2}\right| = 1, |z_1| \neq 3$, then $|z_2|$ = **[EAMCET 2004]**

1) 1 2) 2 3) 3 4) 4

Ans: 1

Sol. $|z_1-3z_2| = |3-z_1\bar{z}_2| \Rightarrow (z_1-3z_2)$

$(\bar{z}_1-3\bar{z}_2) = (3-z_1\bar{z}_2)(3-\bar{z}_1z_2)$

$\Rightarrow z_1\bar{z}_1 + 9z_2\bar{z}_2 = 9+|z_1|^2|z_2|^2$

$\Rightarrow |z_1|^2 + 9|z_2|^2 - 9 - |z_1|^2|z_2|^2 = 0$

$\Rightarrow \left(9-|z_1|^2\right)\left(1-|z_2|^2\right) = 0 \Rightarrow |z_2| = 1$

11. $\sum_{n=0}^{n}\left(\frac{2i}{3}\right)^n$ **[EAMCET 2004]**

1) $\frac{9+6i}{13}$ 2) $\frac{9-6i}{13}$ 3) $9+6i$ 4) $9-6i$

Ans: 1

Sol. $\sum_{n=0}^{n}\left(\frac{2i}{3}\right)^n = \frac{1}{1-\frac{2i}{3}} = \frac{3}{3-2i} = \frac{9+6i}{13}$

12. If the amplitude of $z-2-3i$ is $\frac{\pi}{4}$, then the locus of Z = x + iy is **[EAMCET 2003]**

1) x + y – 1 = 0 2) x – y – 1 = 0 3) x + y + 1= 0 4) x – y + 1 =0

Ans: 4

Sol. $z-2-3i = (x-2)+(y-3)i$

$\tan^{-1}\left(\frac{y-3}{x-2}\right) = \frac{\pi}{4} \Rightarrow x-y+1=0$

13. If ω is a complex cube root of unity, then $225+\left(3\omega+8\omega^2\right)^2+\left(3\omega^2+8\omega\right)^2 = ...$ **[EAMCET 2003]**

1) 72 2) 192 3) 200 4) 248

Ans: 4

Sol. $225+\left(3\omega+8\omega^2\right)^2+\left(3\omega^2+8\omega\right)^2$

$= 225+73\omega^4+96\omega^3+73\omega^2 = 248$

14. If z = x + iy is a complex number satisfying $\left|z+\frac{i}{2}\right|^2 = \left|z-\frac{i}{2}\right|^2$, then the locus of z is

[EAMCET 2002]

1) x-axis 2) y-axis 3) y = x 4) 2y = x

Ans: 1

Sol. Z = x + iy;

$\left|x+iy+\frac{i}{2}\right|^2 = \left|x+iy-\frac{i}{2}\right|^2$

$x^2+\left(y+\frac{1}{2}\right)^2 = x^2+\left(y-\frac{1}{2}\right)^2$

$\Rightarrow y=0 \;\therefore$ x-axis

15. If $z=3+5i$, then $z^3+\bar{z}+198=$ **[EAMCET 2002]**

1) –3-5i 2) – 3 + 5i 3) 3 –5i 4) 3 + 5i

Ans: 4

Sol. $(3+5i)^3+(3-5i)+198 = 3+5i$

16. If $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number and 0 < θ < 2π, then θ = **[EAMCET 2002]**

1) π 2) π/2 3) π/3 4) π/6

Ans: 1

Sol. $\frac{3+2i\sin\theta}{1-2i\sin\theta}\times\frac{1+2i\sin\theta}{1+2i\sin\theta}$

Purely real $\Rightarrow$ Imag. Part = 0

ima . part = $\frac{8i\sin\theta}{1+4\sin^2\theta}=0$

$\sin\theta=0$

$\therefore \theta=\pi$

17. If α is a complex number and b is real number then the equation : $\bar{a}z+a\bar{z}+b=0$ represents a **[EAMCET 2001]**

1) Straight line 2) Parabola 3) Circle 4) Hyperbola

Ans: 1

Sol. Let a = p + iq and z = x + iy

$\bar{a}z+a\bar{z}+b=0$

$\Rightarrow (p-iq)(x+iy)+(p+iq)(x-iy)+b=0$

equating real parts (or) imaginary parts on both sides then the locus of 'z' is straight line.

18. If $\begin{vmatrix}1-i & i\\ 1+2i & -1\end{vmatrix}=x+iy$, then x = **[EAMCET 2001]**

1) 1 2) – 1 3) 2 4) –2

Ans: 1

Sol . $\begin{vmatrix}1-i & i\\ 1+2i & -1\end{vmatrix}=x+iy$

$-1(1-i)-i(1+2i)=x+iy$

$-1+i-i+2=x+iy$

$\therefore x=1$

19. The locus of the point Z in the Argand plane for which $|z+1|^2+|z-1|^2=4$ is a **[EAMCET 2000]**

1) Straight line 2) Pair of straight line 3) Circle 4) Parabola

Ans: 3

Sol. $|z+1|^2+|z-1|^2=4$

$(x+1)^2+y^2+(x-1)^2+y^2=4$

$\therefore\ x^2+y^2=1(\text{circle})$

20. If θ is real, then the modulus of $\frac{1}{(1+\cos\theta)+i\sin\theta}$ is **[EAMCET 2000]**

1) $\frac{1}{2}\sec\frac{\theta}{2}$ 2) $\frac{1}{2}\cos\frac{\theta}{2}$ 3) $\sec\frac{\theta}{2}$ 4) $\sec\frac{-\theta}{2}$

Ans: 1

Sol. $\left|\frac{1}{(1+\cos\theta)+i\sin\theta}\right|$

$=\frac{1}{\sqrt{(1+\cos\theta)^2+\sin^2\theta}}=\frac{1}{\sqrt{2+2\cos\theta}}$

$=\frac{2}{2\cos\theta/2}=\frac{1}{2}\sec\frac{\theta}{2}$

21. If 1, ω, ω^2 are the cube roots of unity, then $(a+b)^2 + (a\omega + b\omega^2)^3 + (a\omega^2 + b\omega)^3 =$

[EAMCET 2000]

1) $a^3 + b^3$ 2) $3(a^3 + b^3)$ 3) $a^3 - b^3$ 4) $a^3 + b^3 + 3ab$

Ans: 2

Sol. $(a+b)^3 + (a\omega + b\omega^2)^3 + (a\omega^2 + b\omega)^3$

$= 3(a+b)(a\omega + b\omega^2)(a\omega^2 + b\omega)$

$= 3(a^3 + b^3)$

22. In the Argand plane the area in square units of the triangle formed by the points $1+i, 1-i, 2i$ is

[EAMCET 2000]

1) 1/2 2) 1 3) $\sqrt{2}$ 4) 2

Ans: 2

Sol. A(1,1) B(1, –1) C(0, 2)

Area ΔABC = $\frac{1}{2}\begin{vmatrix} 1-1 & 1-0 \\ 1+1 & 1-2 \end{vmatrix} = \frac{1}{2}(2) = 1\,sq.unit$

23. If 3 + i is a root of $x^2 + ax + b = 0$, then $a =$ **[EAMCET 2000]**

1) 3 2) – 3 3) 6 4) –6

Ans: 4

Sol. One root is 3 + i then other roots is 3 – i sum of roots = 6 = – a

$\Rightarrow a = -6$

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