CIRCLES

PREVIOUS EAMCET BITS

1. Th equation of the circles which passes through the origin and makes intercepts of length 4 and 8 on the x and y-axis respectively, are : **[EAMCET 2009]**

   1) $x^2+y^2\pm4x\pm8y=0$ 2) $x^2+y^2\pm2x\pm4y=0$

   3) $x^2+y^2\pm8x\pm16y=0$ 4) $x^2+y^2\pm x\pm y=0$

   Ans: 1

Sol. Equations of circles passing through $(0,0),(\pm4,0),(0,\pm8)$ are $x^2+y^2\pm4x\pm8y=0$

2. The locus of centre of a circle which passes through the origin and cuts off a length of 4 units from the line x = 3 is **[EAMCET 2009]**

   1) $y^2+6x=0$ 2) $y^2+6x=13$ 3) $y^2+6x=10$ 4) $x^2+6y=13$

   Ans: 2

Sol. $OP=\sqrt{PM^2+2^2}$

$\Rightarrow\sqrt{x^2+y^2}=\sqrt{(x-3)^2+4}$

$\Rightarrow y^2+6x=13$

3. The diameters of a circle are along 2x + y – 7 = 0 and x + 3y – 11 = 0. Then the equation of this circle, which also passes through (5, 7) is **[EAMCET 2009]**

   1) $x^2+y^2-4x-6y-16=0$ 2) $x^2+y^2-4x-6y-20=0$

   3) $x^2+y^2-4x-6y-12=0$ 4) $x^2+y^2+4x+6y-12=0$

   Ans: 3

Sol. Centre of the circle = point of intersection of diameters = (2, 3) radius = distance between (5, 7) and (2, 3) = 5

4. If the lines 2x – 3y = 5 and 3x – 4y = 7 are two diameters of a circle of radius 7, then the equation of the circle is **[EAMCET 2008]**

   1) $x^2+y^2+2x-4y-47=0$ 2) $x^2+y^2=49$

   3) $x^2+y^2-2x+2y-47=0$ 4) $x^2+y^2=17$

   Ans: 3

Sol. Centre of the circle = Point of intersection of $\{2x-3y=5,3x-4y=7\}=(1,-1)$

Equation of the circle = $(x-1)^2+(y-1)^2=7^2\Rightarrow x^2+y^2-2x+2y-47=0$

5. The inverse of the point (1, 2) with respect to the circle $x^2+y^2-4x-6y+9=0$, is **[EAMCET 2008]**

   1) $\left(1,\frac{1}{2}\right)$ 2) (2, 1) 3) (0, 1) 4) (1, 0)

   Ans: 3

Sol. The polar of (1, 2) is $1x+2y-2(x-1)-3(y+2)+9=0$

$\Rightarrow -x-y+1=0\Rightarrow x+y-1=0$

If (h, k) is the foot of the perpendicular then

$\frac{h-1}{1}=\frac{k-2}{1}=\frac{-(1+2-1)}{1+1}=-1\Rightarrow h=0,k=1\Rightarrow$ Inverse point = (0, 1)

6. If θ is the angle between the tangents from (–1, 0) to the circle $x^2 + y^2 - 5x + 4y - 2 = 0$, then θ = **[EAMCET 2008]**

1) $2\tan^{-1}\left(\frac{7}{4}\right)$ 2) $\tan^{-1}\left(\frac{7}{4}\right)$ 3) $2\cot^{-1}\left(\frac{7}{4}\right)$ 4) $\cot^{-1}\left(\frac{7}{4}\right)$

Ans: 1

Sol. Radius $r = \sqrt{\frac{25}{4} + 4 + 2} = \frac{7}{2}$ $S_{11} = 1 + 0 + 5 + 0 - 2 = 4$

$\tan\frac{\theta}{2} = \frac{r}{\sqrt{S_{11}}} = \frac{7}{2\sqrt{4}} = \frac{7}{4} \Rightarrow \frac{\theta}{2} = \tan^{-1}\frac{7}{4} \Rightarrow \theta = 2\tan^{-1}\frac{7}{4}$

7. Th equation of the circle of radius 3 that lies in the fourth quadrant and touching the lines x = 0 and y = 0 is **[EAMCET 2007]**

1) $x^2 + y^2 - 6x + 6y + 9 = 0$ 2) $x^2 + y^2 - 6x - 6y + 9 = 0$

3) $x^2 + y^2 + 6x - 6y + 9 = 0$ 4) $x^2 + y^2 + 6x + 6y + 9 = 0$

Ans: 1

Sol. Centre = (3, –3) radius = 3

Equation of the circle is $x^2 + y^2 - 6x + 6y + 9 = 0$

8. The inverse point of (1, 2) with respect to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ is **[EAMCET 2007]**

1) (0, 0) 2) (1, 0) 3) (0, 1) 4) (1, 1)

Ans: 3

Sol. Polar of (1, 2) w.r. to given circle is $S_1 = 0 \Rightarrow x + y - 1 = 0$ …….(1)

Foot of the perpendicular from (1, 2) to (1) is (0, 1)

$\therefore$ Inverse point is (0, 1)

9. Observe the following statements : **[EAMCET 2006]**

I) The circle $x^2 + y^2 - 6x - 4y - 7 = 0$ touches y-axis

II) The circle $x^2 + y^2 + 6x + 4y - 7 = 0$ touches x-axis.

Which of the following is a correct statement?

1) Both I and II are true 2) Neither I nor II is true

3) I is true, II is false 4) I is false, II is true

Ans: 2

Sol. For circle $(1) f^2 - c = 4 + 7 = 11 > 0$; For circle $(2) g^2 - c = 4 + 7 = 11 > 0$

Both circles intersect X and Y axis

10. The length of the tangent drawn to the circle $x^2 + y^2 - 2x + 4y - 11 = 0$ from the point (1, 3) is **[EAMCET 2006]**

1) 1 2) 2 3) 3 4) 4

Ans: 3

Sol. Length of tangent = $\sqrt{S_{11}}$

$= \sqrt{1 + 9 - 2 + 12 - 11} = 3$

11. If x – y + 1 = 0 meets the circle $x^2 + y^2 + y - 1 = 0$ at A and B, then the equation of the circle with AB as diameter is **[EAMCET 2005]**

1) $2\left(x^2 + y^2\right) + 3x - y + 1 = 0$ 2) $2\left(x^2 + y^2\right) + 3x - y + 2 = 0$

3) $2\left(x^2+y^2\right)+3x-y+3=0$ 4) $x^2+y^2+3x-y+1=0$

Ans: 1

Sol. $S+\lambda L=0$

$x^2+y^2+y-1+\lambda\left(x-y+1\right)$

Centre $\left(\frac{-\lambda}{2},\left(\frac{\lambda-1}{2}\right)\right)$ lies on $x-y+1=0$

$\therefore\ \lambda=\frac{3}{2}$, Required equation of circle is $2\left(x^2+y^2\right)+3x-y+1=0$

12. If $y=3x$ is a tangent to a circle with centre (1, 1), then the other tangent drawn through (0, 0) to the circle is : **[EAMCET 2005]**

1) $3y=x$ 2) $y=-3x$ 3) $y=2x$ 4) $y=-2x$

Ans: 1

Sol. By verification $r=d$ is true for $3y=x$.

13. If P_1, P_2, P_3 are the perimeters of the three circles $x^2+y^2+8x-6y=0$. $4x^2+4y^2-4x-12y-186=0$ and $x^2+y^2-6x+6y-9=0$ respectively. Then **[EAMCET 2004]**

1) $P_1<P_2<P_3$ 2) $P_1<P_3<P_2$ 3) $P_3<P_2<P_1$ 4) $P_2<P_3<P_1$

Ans: 2

Sol. Perimeter of the circle $=2\pi r$

$r_1=5, r_2=7, r_3=3\sqrt{3}\Rightarrow P_1<P_3<P_2$

14. If the line $3x-2y+6=0$ meets X-axis and Y-axis respectively at A and B, then the equation of the circle with radius AB and centre at A is **[EAMCET 2004]**

1) $x^2+y^2+4x+9=0$ 2) $x^2+y^2+4x-9=0$

3) $x^2+y^2+4x+4=0$ 4) $x^2+y^2+4x-4=0$

Ans: 2

Sol. $A\left(-2,0\right), B=\left(0,3\right)$

$AB=\sqrt{3}$

$\therefore$ Equation of the circle is $\left(x+2\right)^2+y^2=13$

15 .If (1, a), (b, 2) are conjugate points with respect to the circle $x^2+y^2=25$, then $4a+2b=$ **[EAMCET 2004]**

1) 25 2) 50 3) 100 4) 150

Ans: 2

Sol. $S_{12}=0\Rightarrow b+2a=25$

$\Rightarrow 4a+2b=50$

16. If P is a point such that the ratio of the squares of the lengths of the tangents from P to the circles $x^2+y^2+2x-4y-20=0$ and $x^2+y^2-4x+2y-44=0$ is 2 : 3, then the locus of P is a circle with centre **[EAMCET 2003]**

1) $\left(7,-8\right)$ 2) $\left(-7,8\right)$ 3) $\left(7,8\right)$ 4) $\left(-7,-8\right)$

Ans: 2

Sol. $S_{11}:S'_{11}=2:3=3S=2S'$

$\Rightarrow x^2+y^2+14x-16y+28=0$

$\therefore$ Centre = (–7, 8)

17. If $5x - 2y + 10 = 0$ and $12y - 5x + 16 = 0$ are two tangents to a circle, then the radius of the circle is **[EAMCET 2003]**

1) 1 2) 2 3) 4 4) 6

Ans: 1

Sol. Radius = 1/2 (distance between the given lines)

$$= \frac{1}{2}\left(\frac{10+16}{13}\right) = 1$$

18. The equation of the circle of radius 5 and touching the coordinate axes in third quadrant is **[EAMCET 2002]**

1) $(x-5)^2 + (y+5)^2 = 25$ 2) $(x+5)^2 + (y+5)^2 = 25$

3) $(x+4)^2 + (y+4)^2 = 25$ 4) $(x+6)^2 + (y+6)^2 = 25$

Ans: 2

Sol. Centre (–a, –a) radius = a= 5

$\therefore$ $(x+5)^2 + (y+5)^2 = 25$

19. The radius of the larger circle lying in the first quadrant and touching the lines $4x + 3y - 12 = 0$ and the coordinate axes is **[EAMCET 2002]**

1) 5 2) 6 3) 7 4) 8

Ans: 2

Sol. Centre (a, a), radius = a

For distance from centre (a, 0); To line $4x + 3y - 12 = 0$

$$\left|\frac{4a+3a-12}{5}\right| = a \Rightarrow a = 1 \text{to } 6$$

$\therefore$ Large radius = 6

20. The four distinct points (0, 0), (2, 0), (0, –2) and (k, -2) are concylic if k = **[EAMCET 2002]**

1) 2 2) – 2 3) 0 4) 1

Ans: 1

Sol. Equation of circle (0, 0), (a, 0), (0, b) is $x^2 + y^2 - 2 + 2y = 0$

Substituting (k, -2) in equation $\Rightarrow k = 2$

21. A line is at a constant distance C from the origin and meets the coordinate axes in A and B. The locus of the centre of the circle passing through O, A, B is **[EAMCET 2002]**

1) $x^{-2} + y^{-2} = c^{-2}$ 2) $x^{-2} + y^{-2} = 2c^{-2}$ 3) $x^{-2} + y^{-2} = 3c^{-2}$ 4) $x^{-2} + y^{-2} = 4c^{-2}$

Ans: 4

Sol. Let the line be $x\cos\alpha + y\sin\alpha = c$ $\therefore$ $A\left(\frac{c}{\cos\alpha}, 0\right); B\left(0, \frac{c}{\sin\alpha}\right)$

Midpoint of AB is centre of circle passing through O, A, B i.e $\left(\frac{c}{2\cos\alpha}, \frac{c}{2\sin\alpha}\right)$

$\therefore$ Locus of the centre is $x^{-2} + y^{-2} = 4c^{-2}$

22. The equation of the normal to the circle $x^2 + y^2 + 6x + 4y - 3 = 0$ at (1, –2) is **[EAMCET 2001]**

1) $y + 1 = 0$ 2) $y + 2 = 0$ 3) $y + 3 = 0$ 4) $y - 2 = 0$

Ans: 2

Sol. Centre of the circle = (–3, –2) ;

Normal passes through centre

$\therefore$ The normal at (1, –2) is y+ 2 = 0

23. If the polar of a point on the circle $x^2 + y^2 = p^2$ with respect to the circle $x^2 + y^2 = q^2$ touches the circle $x^2 + y^2 = r^2$, then p, q, r are in ……progression **[EAMCET 2001]**

1) Arithmetic 2) Geometric 3) Harmonic 4) Arithmetico Geometric

Ans: 2

Sol. Let (x_1, y_1) is a point on $x^2 + y^2 = p^2 \Rightarrow x_1^2 + y_1^2 = P^2$

The Polar of (x_1, y_1) w.r.t $x^2 + y^2 = q^2$ is $xx_1 + yy_1 = q^2$.....(1)

(1) touches $x^2 + y^2 = r^2$

$\Rightarrow \frac{q^2}{\sqrt{x_1^2 + y_1^2}} = r \Rightarrow q^2 = pr$

24. The centre of the circle touching the y-axis at (0, 3) and making an intercept of 2 units on the positive x-axis is **[EAMCET 2000]**

1) $(10, \sqrt{3})$ 2) $(\sqrt{3}, 10)$ 3) $(\sqrt{10}, 3)$ 4) $(3, \sqrt{10})$

Ans: 3

Sol. The circle touches y-axis at (0, 3)

$\therefore$ f = –3; c = 9

The length of intercept made by the circle on x-axis = $2\sqrt{g^2 - c} = 2 \Rightarrow g = \pm\sqrt{10}$

$\therefore$ centre $= (\sqrt{10}, 3)$

($\because$ lies in the first Quadrant)

25. The slope m of a tangent through the point (7, 1) to the circle $x^2 + y^2 = 25$ satisfies the equation **[EAMCET 2000]**

1) $12m^2 + 7m - 12 = 0$ 2) $16m^2 - 24m + 9 = 0$

3) $12m^2 - 7m - 12 = 0$ 4) $9m^2 + 24m + 16 = 0$

Ans: 3

Sol. The equation of the tangents is $(y - 1) = m(x - 7)$

$\Rightarrow mx - y + (1 - 7m) = 0$.....(1)

(1) touches the circles $x^2 + y^2 = 25$

$\left|\frac{1 - 7m}{\sqrt{m^2 + 1}}\right| = 5 \Rightarrow 12m^2 - 7m - 12 = 0$

26. The number of common tangent that can be drawn to the circles $x^2 + y^2 = 1$ and $x^2 + y^2 - 2x - 6y + 6 = 0$ is **[EAMCET 2000]**

1) 1 2) 2 3) 3 4) 4

Ans: 4

Sol. $C_1'(0,0) C_2(1,3)$, $r_1 = 1; r_2 = 2$

$\overline{C_1C_2} = \sqrt{10} > r_1 + r_2$

✡ ✡ ✡ ✡