CHANGE OF AXES

PREVIOUS EAMCET BITS

1. The transformed equation of $x^2 + y^2 = r^2$ when the axes are rotated through an angle 36° is **[EAMCET 2009]**

   1) $\sqrt{5}X^2 - 4XY + Y^2 = r^2$ 2) $X^2 + 2XY - \sqrt{5}Y^2 = r^2$
   3) $X^2 - Y^2 = r^2$ 4) $X^2 + Y^2 = r^2$

   Ans: 4

Sol. Equation of circle will not change

2. The transformed equation of $3x^2 + 3y^2 + 2xy = 2$ when the coordinate axes are rotated through an angle of 45° is **[EAMCET 2008]**

   1) $X^2 + 2Y^2 = 1$ 2) $2X^2 + Y^2 = 1$ 3) $X^2 + Y^2 = 1$ 4) $X^2 + 3Y^2 = 1$

   Ans: 2

Sol. Let (X, Y) be the new coordinates of (x, y), when the axes are rotated through an angle 45°. Then
$y = X\sin 45° + Y\cos 45° = (X+Y)/\sqrt{2}$ and $x = X\cos 45° - Y\sin 45° = (X-Y)/\sqrt{2}$

The transformed equation is $3\left(\frac{X-Y}{\sqrt{2}}\right)^2 + 3\left(\frac{X+Y}{\sqrt{2}}\right)^2 + 2\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right) = 2$

$\Rightarrow 3\left(X^2 + Y^2\right) + \left(X^2 - Y^2\right) = 2 \Rightarrow 4X^2 + 2Y^2 = 2 \Rightarrow 2X^2 + Y^2 = 1$

3. In order to eliminate the first degree terms from the equation **[EAMCET 2007]**
   $2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$, the point to which origin is to be shifted is

   1) $(1, -3)$ 2) (2, 3) 3) (–2, 3) 4) (1, 3)

   Ans: 3

Sol. $S = 2x^2 + 4xy + 5y^2 - 4x - 22y + 7 = 0$

$\frac{\partial S}{\partial x} = 4x + 4y - 4 = 0;$

$\frac{\partial S}{\partial y} = 4x + 10y - 22 = 0$

(x, y) = (–2, 3)

4. The transformed equation $x^2 + 6xy + 8y^2 = 10$ when the axes are rotated through an angle $\pi/4$ is **[EAMCET 2006]**

   1) $15x^2 - 14xy + 3y^2 = 20$ 2) $15x^2 + 14xy - 3y^2 = 20$
   3) $15x^2 + 14xy + 3y^2 = 20$ 4) $15x^2 - 14xy - 3y^2 = 20$

   Ans: 3

Sol. $\theta = \frac{\pi}{4}$

$x = X\cos\theta - Y\sin\theta = \frac{X-Y}{\sqrt{2}}$

$y = X\sin\theta + Y\cos\theta = \frac{X+Y}{\sqrt{2}}$

transformed equations is

$$\frac{(X-Y)^2}{2}+6\frac{(X-Y)(X+Y)}{2}+8\frac{(X+Y)^2}{2}=10$$

$\Rightarrow 15x^2+14xy+3y^2=20$

5. The coordinate axes are rotated through an angle 135°. If the coordinates of a point P in the new system are known to be (4, –3), then the coordinates of P in the original system are **[EAMCET 2003]**

1) $\left(\frac{1}{\sqrt{2}},\frac{7}{\sqrt{2}}\right)$ 2) $\left(\frac{1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ 3) $\left(\frac{-1}{\sqrt{2}},\frac{-7}{\sqrt{2}}\right)$ 4) $\left(\frac{-1}{\sqrt{2}},\frac{7}{2}\right)$

Ans: 4

Sol. $x=4\cos 135°-(-3)\sin 135°=-\frac{1}{\sqrt{2}}$

$y=4\sin 135°+(-3)\cos 135°=\frac{7}{\sqrt{2}}$

6. If the axes are rotated through an angle 45° in the positive direction without changing the origin, then the coordinates of the point $\left(\sqrt{2},4\right)$ in the old system are **[EAMCET 2002]**

1) $\left(1-2\sqrt{2},1+2\sqrt{2}\right)$ 2) $\left(1+2\sqrt{2},1-2\sqrt{2}\right)$

3) $\left(2\sqrt{2},\sqrt{2}\right)$ 4) $\left(\sqrt{2},\sqrt{2}\right)$

Ans: 1

Sol. $x=X\cos\theta-y\sin\theta$ given $(X,Y)=\left(\sqrt{2},4\right)$

$y=X\sin\theta+Y\cos\theta$ and $\theta=\frac{\pi}{4}$

7. The coordinate axes are rotated about the origin 0 in the counter-clockwise direction through an angle 60°. If p and q are the intercepts made on the new axes by a straight line whose equation referred to the original axes is x + y = 1, then $\frac{1}{p^2}+\frac{1}{q^2}=$ **[EAMCET 2000]**

1) 1 2) 4 3) 6 4) 8

Ans: 1

Sol. The perpendicular distance from origin to x + y = 1 and $\frac{x}{p}+\frac{y}{q}=1$ are equal

$\therefore\ \frac{1}{p^2}+\frac{1}{q^2}=1$

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